# Quantum Theory: From the Viewpoint Of Logic, Probability and Information Theory

## Addressing the “Paradoxes” and “Conundrums

(Mike Ulrey Feb 2015)

“*A creative person is one who can change a problem into a postulate.*” (Goethe paraphrased)

# Goals

## To address puzzles, riddles, weirdness of quantum theory

- Can any of the following be explained by starting with “quantum logic” and/or "quantum probability" (generalizations of classical versions)?
- EPR and "hidden variables". Was EPR really settled by Bell's argument?
- Heisenberg Uncertainty Principle. Just a standard Fourier transform pair relationship in the appropriate context?
- Squared amplitudes of complex numbers represent predicted likelihoods of quantum outcomes. Why? Is there a probabilistic foundational source for the Born Rule?
- Entanglement -- Can this be understood more deeply? Tensor product? Quantum logic? Quantum probability?
- The "measurement" problem, aka wave function "collapse". Is this merely an artifact of using continuous PDE's to model a fundamentally discrete phenomenon?
- Wave-partice duality and the double-slit experiment. So many narratives, which one(s) is (are) right (or at least convincing)? Bohmian mechanics, for example?`

## To offer some alternative formulations to the Hilbert space approach

- Quantum probability
- Quantum logic
- Quantum Bayesianism
- Quantum information

## To connect “abstract” models of quantum theory (e.g. Hilbert space) with “familiar” concepts

- Motion, force, mass, position, energy, momentum, electricity, magnetism, frequency, time

## To make some connections between “theory” (epistemological) and “reality” (ontological)

Example: The act of balance in riding a bicycle.
<u>Epistimological</u>: The theoretical knowledge of the physics involved in maintaining a state of balance.
<u>Ontological</u>: The practical knowledge of how to ride.
*It is important to understand how both are established and grounded.*

- EPR: Maybe Bohr and Einstein were both right, but just talked on different levels. (E. T. Jaynes (Ref 1))
- Einstein posed questions on the ontological level, while Bohr gave answers on the epistemological level .

# Outline

## Observed phemonena

## Lab Experiments

## Thought experiments

## Puzzles, riddles, general weirdness, and “paradoxes”*

## Suggested “solutions” and current status

## Next steps

- A thorough, deep study of EPR --> Bell’s Theorem --> Experiments of Aspect and others --> Possible resolutions
- What do “locality” and “hidden variables” really mean (connect the math to the words!)?
- Quantum probability, quantum logic, quantum Bayesianism, quantum information -- can any of them make the “paradoxes”* disappear?
  - *By “paradox”, I mean a clash with our intuition

# Observed Phenomena

Blackbody radiation: *Classical theory of black-body radiation (Rayleigh-Jeans in 1900 and 1905) does not agree with experiment ("ultra-violet catastrophe"). Planck (1900) invents the idea of frequency quanta (and hence energy quanta via* $E = h\,\nu$*) to model the phenomenon and achieve remarkable agreement with experiment*

http://hyperphysics.phy-astr.gsu.edu/hbase/mod6.html

Photoelectric effect: *Classical theory of photoelectric effect does not agree with experiment. Einstein (1905) adapts Planck's frequency quanta (and hence energy quanta via* $E = h\,\nu$*) to model the effect and achieve remarkable agreement with experiment*

http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html

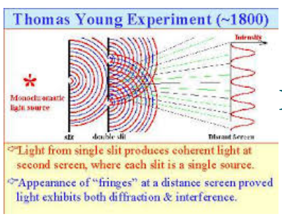
# Lab experiments

## Double-slit experiment (1803): *Light behaves like a wave and a particle*

http://www.youtube.com/watch?v=DfPeprQ7oGc

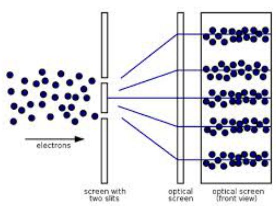

Thomas Young (1773 – 1829)

Championed the wave nature of light. Went against tradition of Newton and the current establishment (e.g., Poisson).



Decisive experiment in 1803. Still took a long time to accept.

Fresnel and Arago convinced Poisson and the French Academy of Sciences a decade later.



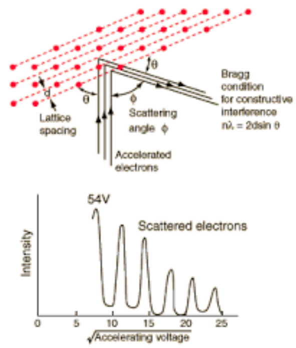
How can this be consistent with corpuscular view of light?

## Zeeman effect (1896): *Spectral lines later explained by quantized energy levels (Zeeman was fired for his work but he and Lorentz won the 1902 Nobel prize)*

## Davison-Germer experiment (1922): *Matter also exhibits wavelike behavior (related to de Broglie's pilot wave hypothesis)*

https://www.youtube.com/watch?v=Ho7K27B_Uu8

C. J. Davisson and L. H. Germer at Bell Labs in 1927 showed that a beam of electrons exhibited diffraction-like beahvior when reflected from a solid crystal (nickel)
Furthermore, the angles of diffraction followed Bragg's formula with a wavelength corresponding to de Broglie's equation $\lambda = h/(m v)$ .




## Stern-Gerlach experiment (1922): *Angular momentum is quantized in space (silver atoms)*

Used silver atoms

Zeeman effect

http://en.wikipedia.org/wiki/Stern% E2 %80 %93 Gerlach_experiment

## Phipps-Taylor experiment (1927): *Stern-Gerlach experiment aopplied to hydrogen atom in the ground state*

http://en.wikipedia.org/wiki/Stern% E2 %80 %93 Gerlach_experiment

## Aspect-Grangier-Roger experiment (1982): *Designed to test Bell's Theorem (inequalities). Experimental realization of EPR* gedankenexperiment, *showing that quantum mechanical predictions are confirmed since Bell inequalities (derived from local, hidden variable assumptions) are violated. Note this showed only that some aspects of QM are not wrong, not that it is the <u>only</u> possible explanation.*

http://en.wikipedia.org/wiki/Alain_Aspect

## Other Bell test experiments (1972-2013): *Also Designed to test Bell's Theorem (inequalities).*

http://en.wikipedia.org/wiki/Bell_test_experiments

# Thought experiments

## EPR (1935): *Reality, locality, hidden variables, and “spooky action at a distance”*

- Einstein and Podolsky and Rosen construct a gedankenexperiment intended to demonstrate shortcomings of the Copenhagen interpretation (Bohr) of QM. Their contention was that QM is incomplete. Note that the term “hidden variables” language was not used explicitly in the original paper, but was brought into the discussion later by John von Neumann and David Bohm. Einstein was not completely happy with the final version of the paper, expressing misgivings to Schrodinger afterwards.

https://www.youtube.com/watch?v=5HJK5tQIT4A
http://en.wikipedia.org/wiki/EPR_paradox

## Bell’s Theorem (1967): *Locality, hidden variables, and QM cannot all be true at the same time*

- Bell’s Theorem: Quantum theory, locality, hidden variables -- pick any two

https://www.youtube.com/watch?v=z-s3q9wlLag

https://www.youtube.com/watch?v=7zfnvGXpy-g

https://www.youtube.com/watch?v=8UxYKN1q5sI

## Delayed-choice (1999): *Wheeler’s double-slit experiment enhancement: Delay asking the particle which slit it went through until after it already went through it*

https://www.youtube.com/watch?v=H6HLjpj4Nt4

http : // vimeo.com/38508798

http://en.wikipedia.org/wiki/Mach% E2 %80 %93 Zehnder_interferometer

# Puzzles, riddles, “paradoxes”, and general weirdness

## The Measurement Problem

- A quantum state is not revealed until a “measurement” is made. As John Stewart Bell has said:
  - “What exactly qualifies some physical systems to play the role of ‘measurer’? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system . . . with a PhD?”

## Schrödinger’s Cat

- Real-size cat figure in the garden of Huttenstrasse 30, Zurich, where Erwin Schrödinger lived 1921 – 1926. Depending on the light conditions, the cat appears either alive or not.

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Import[
 "http://upload.wikimedia.org/wikipedia/commons/thumb/5/5e/Schroedinger_cat.jpg/800px-
   Schroedinger_cat.jpg"]
```

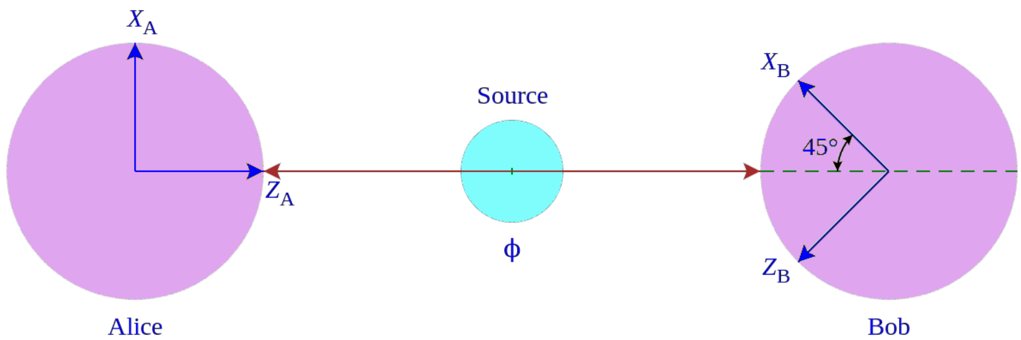
- http://www.youtube.com/watch?v=IOYyCHGWJq4

## The double-slit experiment

- http://www.youtube.com/watch?v=DfPeprQ7oGc
- http://video.mit.edu/watch/thomas-youngs-double-slit-experiment-8432/

## Einstein-Podolsky-Rosen (EPR), Bell’s Theorem, and associated experiments

- “Spooky action at a distance” (Einstein)




- http://www.youtube.com/watch?v=ohfJPboISFo
- https://www.youtube.com/watch?v=5HJK5tQIT4A

## Polarized spin and Bell's Theorem

- http://www.youtube.com/watch?v=7zfnvGXpy-g
- http://www.youtube.com/watch?v=z-s3q9wlLag
- Possible explanation: "Quantum Probability Theory", Hans Maassen, 1998, Section 2 (especially 2.2) <http://www.math.ru.nl/~maassen/lectures/qp.pdf>

## Heisenberg uncertainty principle and incompatible measurements

- http://www.youtube.com/watch?v=ZpwZgOumTrs

## Quantum eraser

- http://www.youtube.com/watch?v=XcZ3jI1Ph7A&list=PLg-OiIIbfPj2RNY2-tYO2JsR9uw7Rw22z&index=4

## Delayed choice experiment

- http://vimeo.com/87175892

## General quantum theory strangeness

- http://www.youtube.com/watch?v=sQfSm6o-KlQ&index=5&list=PLg-OiIIbfPj2RNY2-tYO2JsR9uw7Rw22z
- http://www.youtube.com/watch?v=rkgLzo6qhQg
- http://www.youtube.com/watch?v=zDQH5x7svfg&list=PLg-OiIIbfPj2RNY2-tYO2JsR9uw7Rw22z

- Wave-particle duality -- de Broglie pilot wave and Bohmian mechanics: Has it been given enough respect? Can it be rescued?
- Entanglement -- Can we understand through tensor products alone? Or is there more?
- Particle quantum spin -- what does it mean? Related to entanglement and tensor products...

# Updates on double slit experiment understanding

Robert please review my recollection of your comments and correct where needed.

Some notes based on discussions during my presentation. Especially relevant to the YouTube video http : // www.youtube.com / watch? v = DfPeprQ7oGc

The video illustrates the fact that "observing" one of the slits to see which one a particular electron goes through destroys the interference and recovers the purely particle-like behavior. It shows a mechanical (but charmingly human-like) mechanical eye seemingly passively viewing the slit.

But Robert points out that viewing is never passive, and there must be some form of energy involved coming from the phenomenon to the "observer". In particular, some form of energy like light would have to impinge on the electron for it to be "viewed" in this way, thus modifing the situation.

First, he mentioned a frequency range for electromagnetic radiation consisting of 18 orders of magnitude (I believe). I tried to check just what he might be talking about on Wikipedia, and found a table that shows a range from 3 Hz (ELF) to 300 EHz (gamma rays) (an EHz is 10^18 Hz I believe, from Exa, corresponding to 10^18) and another illustration showing a range from a little below 10^6 Hz (long waves) to 10^19 Hz (gamma rays).

He points out "Robert's Rules of Waves", which to the best of my memory are (please correct me if I'm wrong):

1. For waves to significantly interact, their wavelengths must be roughly within a couple of orders of magnitude of each other.
2. For the double-slit experiment to exhibit interference, the slits must be appropriately spaced, again roughly at wavelength of the particles/waves being fired at the slits.

Consider Wheeler's delayed choice thought experiment in which the "observer" does not look at the slit until after the particle has already gone through, which apparently does not change the results of the experiment. Robert observes

(1) The particle detector must be placed close to the slit. For example, it can't be too far behind the slit since then it essentially becomes the original detector screen, and we already know what happens there (the mysterious switching from interference to particle like behavior when "observed").

(2) It is of small consequence whether one places the detector in front of or behind the slit, since the spacing requirements we have just discussed mean that the waves/particles, slit, and detector are close enough that they must be analyzed as a whole, that is, their wave function is a composite of all the different elements.

(3) As a result, the closeness required for the detector (or an energy source that reflects energy from the wave/particle to a detector) is virtually guaranteed to destroy the interference and cause a reversion to pure "particle" mode.

# Suggested solutions and current status

- Quantum logic
- Quantum probability
- Quantum Bayesianism
- Quantum information
- Other assorted re-thinking of various “givens”, such as: What do “locality” and “hidden variables” really mean mathematically? That is, can Bell’s Theorem be questioned? Not the result, but the interpretation.
- Current status:
  - *We will pursue deeper mathematical investigations into promising and/or interesting alternatives*

# Interpretations: The three major schools of thought

1. Copenhagen interpretation (Bohr) (complementarity -- macroscopic (classical) meets microscopic (quantum) at the time of "measurement")
2. Hidden variables (Bohm)  (based on de Broglie pilot waves that "guide" the particle)
3. Multiple universes (Everett) (no measurement "collapse" -- all possibilities happen at once, creating multiple "realities")

I will be focussed on issues related to #2, such as EPR, Bell's Theorem, Aspect experiment (and others). Spoiler alert: I am still pulling for Einstein's view to win out in the end (in some sense TBD).

# Formulations: Nine major ones plus one other

From “Nine Formulations of Quantum Mechanics”, Styer et al, Oberlin College, 2001. http://susanka.org/HSforQM/

Notes:
1. Attributions and dates are based on original idea, even if it was developed more extensively later by others.
2. These are basically equivalent in their predictions, but provide differing emphases, notation, and methods that may resonate more with some groups than with others, e.g., physicists and mathematicians.
Update based on discussion during the presentation:
3. Robert and others point out that they are not all completly equivalent in that some address different aspects of QM. In particular it was pointed out that “second quantization” refers to quantization of the fields in addition to the “matter”.

- Matrix ( Heisenberg 1925)
- Wave function (Schrodinger 1925 (six months later))
- Path integral (Feynman 1965)
- Phase space (Wigner 1932)
- Density matrix (J von Neumann 1927)
- Second quantization (Dirac 1927 for photons, Jordan and Wigner 1927-8 for fermions)
- Variational (Jordan and Klein 1927)
- Pilot wave (de Broglie 1927 and Bohm 1952)
- Hamilton-Jacobi (Leacock and Padgett 1983)

Others:
- Transactional (Cramer 1980 and 1988)

# Requirements for a new formulation:

- Must produce predictions/explanations that agree with experiment
- Major features of Hilbert space approach can be derived from the axioms of the new formulation
  - Born rule
  - Heisenberg uncertainty principle
  - Non-commutativity and complementary “observables”
  - Entanglement

# Other formulations?

From "Quantum Probability" by Stanley Gudder (1988)

- Quantum logic (Birkoff and von Neumann 1936; Mackey 1963; Bodiou 1964; Varadarajan 1968, 1970; Jauch 1968; Piron 1976; Beltrametti and Cassinelli 1981)
- Algebraic (Jordan, von Neumann and Wigner 1934; Haag and Kastler 1964; Emch 1972; Segal 1974)
- Convexity or operational (Mielnik 1968, 1969; Davies and Lewis 1970; Edwards 1970; Davies 1976; Ludwig 1983, 1985)
- Hidden variables (EPR 1935; Bohm and Bub 1966; Gudder 1970, 1979, 1985; Belifante 1973; Pitowski 1982, 1983)
- Operational statistics (Foulis and Randall 1972, 1978, 1981, 1985; Randall and Foulis 1973, 1981, 1983; Fischer and Ruttiman 1978; Cook 1985)
- Path integral formalism (Feynman 1948, 1949; Schweber 1961; Feynman and Hibbs 1965; Schulman 1981) Note: Gudder included this for its successes, but claims that it does not have a rigorous mathematical foundation, so is not a true axiomatic model.

# The “essence” of QM?

Again, from "Quantum Probability" by Stanley Gudder (1988)

1. QM must be described by a probabilistic theory
2. The probabilities are not always computed “classically”

- “Two outcomes are thought of as interfering if they cannot be distinguished without disturbing the system.”
- In classical probability one computes the probability of an event by summing the probabilities over all outcomes that define that event.
- In quantum probability, one uses a complex-valued (amplitude) function $A$ on the outcome (sample) space to compute probabilities as follows:
  - If the outcomes of an event $E = \{x_1, x_2, \ldots\}$, then $P(E) = \begin{cases} \left|\sum A(x_i)\right|^2 & \text{if the outcomes interfere} \\ \sum \left|A(x_i)\right|^2 & \text{if the outcomes do } \textit{not} \text{ interfere} \end{cases}$

The second case is essentially classical probability. In the first case, the sum decomposes into two parts, the first part being the classical part, and the second part (the crossterms) corresponds to constructive or destructive interference, as in a wave. For example:

- $|A(x_1) + A(x_2)|^2 = |A(x_1)|^2 + |A(x_2)|^2 + A(x_1)A(x_2)^* + A(x_2)^* A(x_1)$
- Adjoin an amplitude function axiom to the other axioms of a system? May be too restrictive.
- Better to adjoin the properties of an amplitude function as additional axioms (*I admit I don't understand at the moment what he means by this*).

# Potential Topics from Gudder's Book

Chapter titles:

- Classical Probability Theory ☺
- Traditional Quantum Mechanics
- Operational Statistics ☺
- Amplitudes and Transition Amplitudes
- Generalized Probability Spaces
- Probability Manifolds ☺
- Discrete Quantum Mechanics ☺

# A Simple Example

From SIAM Review June 2009 “A Discrete Invitation to Quantum Filtering and Control”
Let $H$ be a Hilbert space with dimension 2. Fix some state $\psi \in H$ and let $X = X^*$ be a self-adjoint operator on $H$. The set $C = \{X\}$ satisfies the conditions of the spectral theorem so there is an orthonormal basis in $H$ such that we can express $X$ and $\psi$ in this basis by

$X = \begin{pmatrix} x_1 & 0 \\ 0 & x_2 \end{pmatrix}$ and $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$.

We can now interpret $X$ as a random variable on some probability space. Introduce $\Omega = \{1, 2\}$, the map $x : \Omega \to \boldsymbol{R}$, by $x(i) = x_i$, and the probability measure $P(\{i\}) = |\psi_i|^2$ for $i = 1, 2$. Then

$P(x = x_i) = |\psi_i|^2$ for $i = 1, 2$, and

$\langle \psi, X \psi \rangle = x_1 |\psi_1|^2 + x_2 |\psi_2|^2 = E_P(x)$ (the expected value of the random variable $x$ under the probability measure $P$)

A probability space is a triple $(\Omega, \mathcal{F}, P)$ where $\Omega$ is the sample space, or set of possible outcomes, $\mathcal{F}$ is a $\sigma$-algebra of subsets of $\Omega$ (the collection of (measurable) events), and $P$ is the probability measure.

This can be easily extended to more general sets $C$ to represent a set of *commuting* observables on a classical probability space and thus compute joint statistics. What we *cannot* do is simultaneously interpret two *non-commuting* observables as random variables on the *same* probability space.

However, the authors reserve the right to show later that they can estimate the statistics of observables they did not choose to measure, even if they do not commute with each other, as long as they commute with observables they did choose to measure.

# A Short Random Sample of Papers (and a book) with Interesting Hypotheses

“Quantum Probability Spaces”, Stanley Gudder, AMS, 1967?.
Premise is that a suitable generalization of classical probability serves as a framework from which quantum mechanical phenomena can be derived in a transparent fashion.

"Quantum Probability", Stanley Gudder, Academic Press, 1988.
Emphasizes the axiomatic method in developing extensions to classical logic and probability theory that encompass the phenomena of QM (mostly). After brief reviews of classical probability and the Hilbert space QM formulation, Gudder looks at quantum logic, experimental propositions, algebraic approach, convexity or operational approach, hidden variables approach, and the operational statistics approach. Very detailed and mathematical with lots of food for thought and good references.

“Quantum Theory from Five Reasonable Axioms”, Lucien Hardy, February 1, 2008
Show quantum (and classical) probabilty theory can be derived from five simple axioms. Only the first four are necessary for classical probability and it is the fifth which gets us to quantum theory. Axiom 5 requires the existence of continuous reversible transformations between states.

“Why Does Nonlocality Support Bohm’s Theory?”, Jean Bricmont, April 2010.
Some quotes by various physicists on EPR, Bohm, and Bell. Not sure of the conclusions.

“QBism, the Perimeter of Quantum Bayesianism”, Christopher Fuchs, Mar 2010.
A lot to absorb. There are some hints that “symmetric informationally complete” (SIC -- pronounced “seek”) positive-operator-valued measure may be the key to deriving all of QM from a Bayesian probability viewpoint, but there are some outstanding issues to be resolved. In other words, still more work to do.

“General Fidelity Limit for Quantum Channels”, Fuchs, Jozsa, Schumacher, March 1996.
Technical paper on quantum coding with fidelity criterion. Establishes a general quantum noiseless coding theorem.

# YouTube videos to watch and papers to read

| Issue | Description | Possible res |
|---|---|---|
| Double slit experiment | http : // www.youtube.com / watch ? v = DfPeprQ7oGc | □ |
| Measurement problem | http : // www.youtube.com / watch ? v = ohfJPboISFo | □ |
| Schrodinger ' s cat | http : // www.youtube.com / watch ? v = IOYyCHGWJq4 | □ |
| Bell ' s Theorem | http : // www.youtube.com / watch ? v = 7 zfnvGXpy – g | □ |
| EPR | http : // www.youtube.com / watch ? v = ohfJPboISFo | □ |
| Heisenberg uncertainty | https : // www.youtube.com / watch ? v = TQKELOE9eY4 | □ |
| Delayed choice | http : // vimeo.com / 38 508 798 | □ |
| Quantum eraser | https : // www.youtube.com / watch ? v = sQfSm6o – KlQ | □ |
| General QT "strangeness" | □ | □ |
| Wave – Particle Duality | https : // www.youtube.com / watch ? v = Hk3fgjHNQ2Q | □ |

# EPR Resolutions?

## Bohmian mechanics or Pilot-Wave Theory: Provides a deterministic alternative to traditional quantum theory. The problem is “contexuality” when it comes to spin. Bohmian mechanics works fine for position, momentum, etc. So maybe our concept of spin as a real “property” of a particle is wrong?

https://www.youtube.com/watch?v=rbRVnC92sMs

https://www.youtube.com/watch?v=Qz4CHI_W-TA

# Classical and Quantum Logic

"On Quantum vs. Classical Probability", Jochen Rau, April 28, 2009
Comparison of classical and quantum logic.

| Correspondence between classical and quantum logic | | | |
|---|---|---|---|
| Classical concept | Quantum Analog | Generic Name | Symbol |
| Sample space | Hilbert space | Proposition system | __ |
| Subset | Subspace | Hypothesis, proposition | a,b,x,y |
| Element | 1−dim subspace (ray) | Most accurate hypothesis | e,f |
| Empty set | Zero | Absurd hypothesis | $\phi$ |
| Disjointedness | Orthogonality | Contradiction, exclusion | $\perp$ |
| Set inclusion | Embedding | Implication, refinement | $\subseteq$ |
| Set complement | Orthogonal complement | Complement, refinement | \ |
| Cardinality | Dimension | Granularity | d |

# Spin -- Dirac combines quantum theory and special relativity

Relativistic Hamiltonian: *Dirac makes bold assumptions, finds that scalar linear equations must be replaced by corresponding equations in 4-vectors (space and time).  and invents the 4 x 4 Dirac matrices*

Existence of positron is predicted using the allowed solution with negative total energy (confirmed 1932 by Anderson)

Dirac's theory gives rise to concept of spin. Not quite the same as classical spin. See for example....

https://www.youtube.com/watch?v=v1_-LsQLwkA
https://www.youtube.com/watch?v=28Xe4FCCjt4
https://www.youtube.com/watch?v=W-_4zOYTIVU

# Multi-Particle States and Tensor Products

## From “Multi-Particle States and Tensor Products” (B. Zwiebach MIT Course 2013)

http://ocw.mit.edu/courses/physics/8-05-quantum-physics-ii-fall-2013/lecture-notes/MIT8_05F13_Chap _ 08.pdf

“The tensor product $V \otimes W$ is the complex vector space of states of a two-particle system!”

This shows clearly that a general state of the two-particle system cannot be described by stating the state of the first particle and the state of the second particle. The above superpositions give rise to entangled states. An entangled state of the two particles is one that, roughly, cannot be disentangled into separate states of each of the particles. We will make this precise soon.

# The Goal and the Plan

## The Goal:

## Explore whether or not Bell's Theorem and the Bell Test Experiments (e.g., Aspect et al) are the final word on the EPR thought experiment. For example, how are the terms "local" and "hidden" expressed in the math? Are these concepts too tied up with classical probability, and would our conclusions change if we express them using quantum probability?

## The Plan: Learn about --

Tensors and tensor products. These are fundamental to the mathematization of entangled states. As a bonus, they will help you understand general relativity.

Classical probability: A very quick tour of probability spaces, $\sigma$-algebras of events, random variables, conditional probability, expectation, weak and strong laws of large numbers, the Central Limit Theorem, Baye's Theorem and Bayesians vs Frequentists, exchangeble random variables and deFinetti's theorem.

Simpson's paradox: A very surprising, non-intuitive phenomenon in classical probability and statistics. The classical probability analogue of quantum entanglement.

Quantum probability: Why classical probability is inadequate for QM and what needs to be added in the way of concepts and axioms. Will cover some but not all of the topics in Steven Gudder's "Quantum Probability" text. Emphasis on "operational statistics" and "generalized probability spaces". Can QM be expressed in completely probabilistic/statistical/informational language, without (explicit) recourse to the classical Dirac-von Neumann Hilbert space approach?

Quantum Bayesianism: What is it? What is its relation to quantum probability? Does it change our perspective on any of the "paradoxes" of QM?

Quantum logic: If we have time, a very brief introduction and how it extends classical logic to include QM, similar to the way quantum probability extends classical probability to include QM.

# References


## Quantum logic, quantum probability, quantum information, and quantum Bayesianism

1. “Quantum Probability Theory”, Hans Maassen, Jan 2003, Section 2 (especially 2.2) <http://www.math.ru.nl/~maassen/lectures/qp.pdf>
2. “An Introduction to Quantum Probability, Quantum Mechanics, and Quantum Computation”, Greg Kuperberg, UC Davis, Oct 8, 2007
3. “Quantum Probability”, Stanley Gudder, Academic Press, 1988.
4. “Quantum Probability Spaces”, Stanley Gudder, American Mathematical Society, 1968.
5. “A Discrete Invitation to Quantum Filtering andd Feedback Control”, Bouten, Handel, and James, SIAM Review, Vol 51, No 2, June 2009.
6. “Unknown Quantum States: The Quantum de Finetti Representation”, Caves, Fuchs, and Shack, April 2001.
7. “Quantum Mechanics as a Theory of Probability”, I. Pitowsky, Feb 1, 2008.
8. “QBism, the Perimeter of Quantum Bayesianism”, C. Fuchs, Mar 26, 2010.
9. “Quantum Mechanics as Quantum Information, Mostly”, C. Fuchs, Date?.
10. “Quantum Theory from Five Reasonable Axioms”, Lucien Hardy, Feb 1, 2008.

## Tensors and Tensor Products

1. “Tensor Products”, Keith Conrad <http://www.math.uconn.edu/~kconrad/blurbs/linmultialg/tensorprod.pdf>
2. “Calculus on Manifolds”, Michael Spivak, Addison-Wesley, 1965
3. “An Introduction to Linear Algebra and Tensors”, M. A. Akivis and V. V. Goldberg, (Translated from the Russian by Richard A. Silverman), Dover, 1972
4. “Chapter 9 Multilinear Algebra”, Smolinsky https://www.math.lsu.edu/~lawson/Chapter9.pdf
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https://www.youtube.com/watch?v=3_BzTMeV4HI
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3. (Stern-Gerlach and Phipps-Taylor experiments) http://en.wikipedia.org/wiki/Stern% E2 %80 %93 Gerlach_experiment
4. (Black-body radiation) http://hyperphysics.phy-astr.gsu.edu/hbase/mod6.html
5. (Photoelctric effect) http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html
6. (Particle spin -- the first one was really helpful to me) https://www.youtube.com/watch?v=v1_-LsQLwkA
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http://www.youtube.com/watch?v=e0eJXttPRZI

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# Initialization

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SetOptions[EvaluationNotebook[], ShowGroupOpener → True]
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