Geometry, Reality and the World of Sensation

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Introduction

Over the last few millennia, since the invention of the modern idea of axioms and logical consequence as used by the Greeks, we have had an evolving relationship between the mathematical subject of geometry, the universe that contains us and our sensations that provide us certain types of information about things external to individual humans, in time.

Geometry

The study of undefined entities using a particular short list of assumed facts about them and subsequent knowledge derived by rules of inference: Logic.

Philosophers use the words ontology and epistemology to talk about the objects of study and the means by which we acquire knowledge of these objects.

The Universe

This is the "meta" world.

In Mathematics it is the world that gives meaning to the axioms. We already understand that $2 + 3 = 5$ before we codify it all as "the ring of integers."

In Physics it is the actual objects in the physical world, though over the centuries the presumed nature of these objects has changed.

SENSATIONS

Sensations provide us certain types of information about things external to individual humans, in time.

Sensations can be used to form the actual objects of study, or they can be used to acquire knowledge of these objects.

Plato and Socrates

Plato (424-348 BCE), the most classic of classical philosophers, founded the Academy in Athens which was the first European institution of higher learning on record. Apparently, we have essentially everything he wrote.

Plato makes it clear in his “Apology of Socrates”, that he was a follower of Socrates. In that dialogue, Socrates is presented as mentioning Plato by name as one of those youths close enough to him to have been “corrupted.”

Plato and Socrates

According to Socrates (470-399 BCE) (in other dialogues of Plato) physical objects and physical events are “distorted shadows” of their ideal or perfect Forms.

There are no Circles in the world, nor is there Love or Justice. Socrates is said to think that perfect Justice exists and his own trial was a corrupt instance. There is an ideal “Red” and all red things in the world are instantiations of it.

The Nature of Forms

Here is a passage from Plato's Republic, a part of a dialog in which Socrates baffles a student.

> *But someone who, to take the opposite case, believes in the beautiful itself, can see both it and the things that participate in it and doesn't believe that the participants are it or that it itself is the participants–is he living in a dream or is he awake?*

Aristotle (384-322 BCE) codified and amplified our understanding of logical consequence (among many other things, of course).

He claimed that Plato took from the Pythagoreans (Pythagorus of Samos lived roughly 570-495 BCE) the idea that mathematics and, generally speaking, abstract thinking can be taken as a secure basis for conclusions about Archetypal Entities in areas beyond mathematics.

The idea here is that the process of extracting Axioms from observations about corrupt instances of lines and circles in the world and thereby creating Theorems about objects in the Perfect Ideal world to which the visible versions refer offered the cleanest possible examples of how to obtain knowledge about Perfect Ideal Things more generally, such as Virtue, Beauty, the Good and so on.

Mathematics was to be the “test case” for perceiving Truths about these other Forms.

Here is the reason that the Philosopher-Kings in Plato’s utopia study mathematics for many years before entering politics.

So to the Greeks, there is something innate in humans that let's us see aspects of lines and circles and other objects which exist in an ideal world: perfect, unchanging, out of time. We have a mysterious natural affinity for these Forms, but cannot see them directly.

Mathematics is to be the conduit, our window into the ideal world.

Euclid

Euclid of Alexandria (323-283 BCE) is reported to have organized the geometrical Axioms and Theorems into "The Elements", which was the heart and gold-standard of mathematical thought in Europe, the Eastern Empire and the Islamic World for over 2000 years, known and used by everyone who studied mathematics for all that time.

It WAS mathematics, unquestionably the most widely read book during all that time. (The Bible and The Koran were not generally available to common folk.)

Objects of Study: the Forms of Circle, Line, etc.

Means of Acquiring Knowledge: Mathematics/Logic

The terms “a priori knowledge” and “a posteriori knowledge” (in their Greek form) are mentioned here, expanded upon endlessly by philosophers, culminating in Kant’s (1781) “Critique of Pure Reason” to include a Synthetic-Analytic distinction.

The First Four of Euclid's Postulates:

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.

The Fifth

5. If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.

This postulate in its contra-positive form, is known as the parallel postulate.

Mathematicians were quite sure that the Forms must have the first four qualities, no one doubted that the fifth was true . . . but many had an intuition that the fifth was not an independent axiom and could be proved from the other four.

Many incorrect "proofs" of this appeared by famous mathematicians

— by Ptolemy (100-170), Proclus (Byzantine, last of the "classic" Greek philosophers, 412-485), Egyptian Ibn al-Haytham (born in Basra, lived 965-1040), the Persian Omar Khayyám (1034-1131) and many others up to Swiss mathematician Johann Lambert (1728-1777).

A few words about these folks:

al-Haytham is usually regarded as the “father” of the modern notion of scientific progress: emphasized experimental data, reproducability. (optics)

Khayyám assembled many of the facts of algebra in more-or-less modern form, and explored its relation to geometry,

Islamic Progress, Preserving and Extending Greek Thought

During the chaotic period in Europe after the fall of the Roman Empire mathematics was kept alive in Constantinople (Byzantium, the Eastern Roman Empire) and, after a bit of maturation, in a highly cultured Islamic world.

During the crusades the (comparatively barbaric) European aristocracy was in contact with and impressed by Islamic philosophers and mathematicians, as well as people whose practices resemble modern scientists.

The Catholic Church thought the status quo in scientific literacy—that the hand of the deity could be seen second by second controlling everything in detail—was more beneficial to civil order.

It was a fight the Church gradually lost.

Caliph Harun al-Rashid (786-809) created the House of Wisdom in Baghdad. He collected scholars from around the world to translate all of the world's classical knowledge into Arabic.

In addition to the works of the Greeks (many translated to Latin through Arabic) the works of the Persians al-Khwārizmī (780-850) and Avicenna (980-1037), al-Biruni (973-1048) became known in Europe.

al-Khwārizmī is known as the inventor of algebra, building on the work started by Diophantus of Alexandria (circa 210-290).

al-Biruni contributed to development of experimental science, combined static and dynamic ideas into mechanics and hydrodynamics

The works of the brilliant Jewish (Egyptian) philosopher Maimonides (1135-1204), the Andalusian Averröes (1126-1198, the Latinized name of ′Abū l-Walīd Muhammad Ibn ′Ahmad Ibn Rushd) and many others became known in Europe, translated to Latin from Arabic.

Famous teachers such as Peter Abelard (1079-1142) popularized newly re-discovered (in Europe) Aristotle and the Islamic world's advances at his schools in Paris and elsewhere.

Public knowledge of the pathetically barbaric conditions at home versus the prosperous intellectually stimulating conditions of Islamic cities could not be contained as crusaders returned.

Heretics of *many* types arose, many of whom claimed that the deity did NOT manage minutiae but had set the laws of the world in motion at the moment of creation and it was the job of philosophers, whose minds and inquisitiveness were also created by the deity, to discover the patterns embedded in our world.

Burning supposed heretics by the hundreds didn’t really stop these ideas.

Adelard of Bath

As an example of the ferment caused by contact with eastern civilization, we have the European proto-scientist Adelard ("Quaestiones Naturales", 1080-1152) as told by Will Durant in his "The Age of Faith:"

> *Adelard, after studying in many Moslem countries, returned to England and wrote (c. 1130) a long dialogue, "Quaestiones Naturales", covering many sciences. It begins Platonically by describing Adelard's reunion with his friends. He asks about the state of affairs in England; he is told that the kings make war, judges take bribes, prelates drink too much, all promises are broken, all friends are envious. He accepts this as a genial summary of the natural and unchangeable condition of things, and proposes to forget it.*

His nephew inquires what has Adelard learned among the Moslems? He expresses a general preference for Arabic as against Christian science; they challenge him; and his replies constitute an interesting selection from all the sciences of the age. He inveighs against the bondage of tradition and authority. "I learned from my Arabian masters under the leading of reason; you, however, captivated by…authority, follow your halter. For what else should authority be called than a halter?" Those who are now counted as authorities gained their reputation by following reason, not authority. "Therefore," he tells his nephew, "if you want to hear anything more from me, give and take reason…"

"Nothing is surer than reason…nothing is falser than the senses." Though Adelard relies too confidently on deductive reasoning, he gives some interesting replies. Asked how the earth is upheld in space, he answers that the center and the bottom are the same. How far would a stone fall if dropped into a hole bored through the center of the earth to the other side? He answers, Only to the center of the earth. He states clearly the indestructibility of matter, and argues that universal continuity makes a vacuum impossible. All in all, Adelard is a brilliant proof of the awakening intellect in Christian Europe in the twelfth century. He was enthusiastic about the possibilities of science, and proudly calls his age-the age of Abelard-modernus, " the climax of all history."

Ultimately, increasing wealth and broader education in the Renaissance (1300-1600), and exposure through trade and travel to the Islamic and Eastern world, overwhelmed the ability of the Church to stem the tide. The European Dark Ages (they were never dark in Islam and Asia) came, unevenly and over centuries, to an end.

With the assistance of the East, and through the natural cycles of rise and fall (and the obliteration of huge swaths of the Islamic world outside of North Africa from the Mongol invasions) an "educated" Europe rose with its own additions. Dozens of Universities took this intellectual treasure from the East and moved on.

The tools needed for a Newton to play his role were assembled.

But to get back to our geometric story …

In 1829 Nikolai Ivanovich Lobachevsky and (1831) János Bolyai described geometries in which the parallel postulate was false.

Gauss claimed (and who could doubt him?) that he had been proving results about these geometries for 35 years.

Eugenio Beltrami in 1868 finally proved that these other geometries (hyperbolic and elliptic) were actually consistent establishing the independence of the fifth from the first four axioms, settling a 2000 year-old question.

So that is the end of that thread, but remember that story seems to be only parenthetically interesting to us. Greek mathematics was all about proving things about Forms: they were only marginally interested in things in our "corporeal" world which, to them, was not real but a result of defects in our being and nature.

The world of the senses was not real to them.

So let's move on to The Scientists.

Setting The Stage

al-Khwārizmī's algebra was expanded-upon and widely understood.

Fibonacci (Leonardo Bonacci 1170-1250) —Robert Grosseteste (1175-1253) —Roger Bacon (1219-1292) — Albertus Magnus (1200-1280) —Duns Scotus (1266-1308) —Theodoric of Freiberg (1250-1310) — William of Ockham (1285-1350) —Jean Buridan (1300-1358)

da Vinci to Newton

Leonardo da Vinci (1452-1519) —Nicolaus Copernicus (1473-1543) (always included on lists like this…?) —Gerolamo Cardano (1501-1576) —Francis Bacon (1561-1626) —Galileo Galilei (1564-1642) —Johannes Kepler (1571-1630)

René Descartes (1596-1650) invented the concept of coordinate system as we know it, the Cartesian coordinate system, “algebraizing” geometry into analytic geometry.

Christiaan Huygens (1629-1695) —Robert Hook (1635-1703)

After all this stage-setting Isaac Newton (1642-1726) created his version of Calculus and, subsequently, what is now known as Classical Mechanics.

He was a key participant in a paradigm shift.

The Scientific Revolution

In this thread, the World of the Senses *is* the world.

Reality is not an out-of-space-and-time universe of Forms. Reality is nothing more than what you can sense and measure.

The interesting things in this world, the things that catch the eye, are things that change. Scientific Theories propose a cause of this change, and if correct that thing becomes predictable.

If it is predictable it can be used.

Scientific Theories are encapsulated in Differential Equations, which propose a cause of changes in things that can be measured in terms of changes in *other* things that can be measured.

The Scientific Revolution proposed that names be given to these real things, things that can be measured, and knowledge of these things is obtained by 1) creating proposals for differential equations which these things must satisfy and 2) verifying through observation that the predictions are correct.

A new ontology. A new epistemology.

Unobservable things are not found anywhere.

One thing that was temporarily lost was classical mathematical rigor, though these defects have been repaired over the centuries.

Newton and Leibnitz and the Bernoulli's (and . . .) did not care. These folks just went about answering question after question that had been in the air for thousands of years.

Bishop Berkeley's (1685-1753) snarky critique, including a reference to "the ghosts of departed quantities" regarding infinitesimals which could be divided by other infinitesimals to get measurable things (if done *just right*) couldn't, in the end, stand up to this success.

And this new framework has been fantastically successful.

It does not apply to everything humans want to know. Nor does it tell any scientist *how* to invent these Scientific Theories, these differential equations.

What it *does* do is just one simple but *completely new*, revolutionary little thing.

It tells us when we are wrong, and provides a social imperative requiring the most ardent believer in a theory to give it up when it is shown to be wrong.

A young genius is unlikely to waste a lifetime on a beautiful wrong Scientific Theory.

That genius will find the theory to be wrong and move on to try to find a better theory.

That's it. That is the key to electricity, to space flight, to medical advances, to cars, to iPhones to …7.125 Billion residents on this small planet, for better or worse.

We have a way of knowing when we're wrong …*on Scientific matters.*

It works!

The reasoning behind the calculations in Newtonian Mechanics involves, implicitly, the ideas of vectors and coordinate space—though Newton himself in his published work often used laborious calculations involving intricate geometrical constructions.

Real progress was made, big jumps in understanding occurred, whenever notation and linearization techniques improved.

Leibnitz: Determinants 1693

Cramer: Cramer's Rule 1750

Gauss: Gaussian Elimination, FFT (Ceres) 1809

Caspar Wessel: (1745-1818) was the first to use complex numbers ("On the Analytical Representation of Direction" 1799) as tools to handle rotations.

Hamilton: Quaternions 1843

Graßmann: 1844 Grassmann Algebra

Sylvester: matrix 1848

Cayley: 1853, Matrix Multiplication and Inverses

Hüseyin Tevfik Pasha wrote the book (1882) "Linear Algebra".

Giuseppe Peano (1888) put together the main ideas of Vector Spaces in a palatable and (mostly) modern form.

So during this time Reality was assumed *to be* the world of our senses, possibly augmented with microscopes or galvanometers and so on, but the normal human interpretation of the world around was not in doubt. Real things were measurable things.

The world is to be taken literally. It imposes real things on you directly through sensation.

The objects of mathematics are not real in this way.

This is in direct contrast to the way mathematicians think about their work. All working mathematicians believe that the things they work on are real. A non-commutative ring, studied for 20 years, is *more* real than a physical chair.

But…what *is* the second infinite cardinal number? What type of thing, if any?

We have a puzzle. What *are* abstract objects-of-the-mind?

1) A pattern of electric potentials and chemical markers in a wet computer

2) The set of all instances of "three-ness" is the number 3

3) An arrangement of symbols that obeys certain rules

4) Nothing. An elaborate hallucination. A fantasy.

It is hard to get away from the Greek idea of Forms when you spend your life thinking about an abstract object.

Putting that aside for now, Mathematicians began pecking away at the edge of the "The Only Real Things are the Measurable Things" dogma. (Hilbert: Existence is guaranteed by consistency.)

The Vienna Circle

An influential group of philosophers and scientists who met from 1924 to 1936 at the University of Vienna, initially organized by Moritz Schlick. Various members continued to publish in an associated journal "International Encyclopedia of Unified Science" 1938-1969

Generally followers of Ernst Mach (1838-1916), who criticized Newton's theories of space and time, anticipating Einstein's theory of relativity.

Influenced more directly by David Hilbert, Gottlob Frege, Bertrand Russell, Henri Poincaré, Albert Einstein

Sympathetic to the ideas of David Hume (empiricism, skepticism, and naturalism "A Treatise of Human Nature", "An Enquiry Concerning Human Understanding" 1711-1776) and Immanuel Kant ("Critique of Pure Reason", "Critique of Practical Reason", 1724-1804)

HUME

Hume's empiricist approach places him in line with Francis Bacon (1561-1626), John Locke (1632-1704), Thomas Hobbes (1588-1679), George Berkeley (1685-1753)

He argued that our **entire** conception of the world was based on experience and was skeptical of the idea that we had access outside of experience to any absolute truths about the world at all. How can we justify the idea that nature will *continue* any percieved pattern . . .

In A Treatise of Human Nature,

> *Tis evident, that all the sciences have a relation, more or less, to human nature. . . Even Mathematics, Natural Philosophy, and Natural Religion, are in some measure dependent on the science of Man.*

Hume

There are two types of statements about the world:

Statements about ideas. These are analytic, necessary statements that are knowable a priori

Statements about the world. These are synthetic, contingent, and knowable a posteriori.

Hume

In "An Inquiry into Human Understanding"

> *All the objects of human reason or enquiry may naturally be divided into two kinds, to wit, Relations of Ideas, and Matters of fact. Of the first kind are the sciences of Geometry, Algebra, and Arithmetic ... discoverable by the mere operation of thought ... Matters of fact, which are the second object of human reason, are not ascertained in the same manner; nor is our evidence of their truth, however great, of a like nature with the foregoing.*

HUME

Also from "An Inquiry into Human Understanding"

> *If we take in our hand any volume; of divinity or school metaphysics, for instance; let us ask, Does it contain any abstract reasoning concerning quantity or number? No. Does it contain any experimental reasoning concerning matter of fact and existence? No. Commit it then to the flames: for it can contain nothing but sophistry and illusion.*

Kant

Kant held that our experiences (and therefore the things we think are important) are structured by necessary features of our minds: space, time, cause, effect.

He was something of a scientist himself:

> *Kant pointed out in the middle of last century, what had not previously been discovered by mathematicians or physical astronomers, that the frictional resistance against tidal currents on the earth's surface must cause a diminution of the earth's rotational speed. This immense discovery in Natural Philosophy seems to have attracted little attention—indeed to have passed quite unnoticed among mathematicians, and astronomers, and naturalists, until about 1840, when the doctrine of energy began to be taken to heart.*

Lord Kelvin, physicist, 1897

Kant

Are there synthetic a priori propositions?

The *interesting* part of Critique of Pure Reason (in my view) is devoted to examining whether and how knowledge of synthetic a priori propositions is possible.

Some Core Members of the Vienna Circle: Hans Hahn, Otto Neurath, Rudolf Carnap, Richard von Mises, Kurt Gödel, Alfred Tarski, Hans Reichenbach, Carl Gustav Hempel, Willard Van Orman Quine

Associated as contributors and critics: Ludwig Wittgenstein and Karl Popper and Thomas Kuhn ("The Structure of Scientific Revolutions" 1962).

Logical Positivism: Attempt to create foundations for the natural and social sciences, logic and mathematics, the modernization of empiricism by advances in logic, the search for an empiricist criterion of meaning, the critique of metaphysics and the unification of the sciences…

Back to Geometry

Leonhard Euler (1707-1783) studied curvature on surfaces.

Gauss (1777-1855) published in 1827 “Disquisitiones Generales Circa Superficies Curvas” and set the stage for physical applications of these more general geometries, a study that resulted from his contract to improve the the geodetic description of Hanover.

This little book contains Gauss’s “Theorema Egregium” which can be translated from the Latin roughly as “Totally Awesome Theorem” and states in modern language “The Gaussian curvature of a surface is invariant under local isometry.”

Theorema Egregium

From Spivak, A Comprehensive Introduction to Differential Geometry Volume II:

> *If* $f, g\colon \mathcal{M} \to \mathbb{R}^3$ *are two embeddings (or even immersions) such that* $f^*\langle\cdot,\cdot\rangle = g^*\langle\cdot,\cdot\rangle$*, then the Gaussian curvature of* $f(\mathcal{M})$ *at* $f(p)$ *is the same as the Gaussian curvature of* $g(\mathcal{M})$ *at* $g(p)$*.*

Quote from Gauss:

> *Thus the formula of the preceding article leads itself to the remarkable Theorem. If a curved surface is developed upon any other surface whatever, the measure of curvature in each point remains unchanged.*

Frenet-Serret formulas describe moving frames on curves in space, describing the derivatives of the tangent, normal, and binormal unit vectors in terms of each other (about 1850).

The Darboux frame or trièdre mobile (Jean Gaston Darboux about 1890) generalized this to surfaces

Élie Cartan extended this to more general homogeneous spaces in his moving frames, the repère mobile, 1935.

Gauss's student Bernhard Riemann (1826-1866) gave a series of talks, prompted by Gauss and in support of Riemann's Habilitationsschrift, on the foundations of geometry. Riemann developed his theory of higher dimensions and in 1854 gave his famous lecture at Göttingen "On the hypotheses which underlie geometry".

Gauss, in the audience, was said to be astonished by the depth of the ideas presented but the results were not generally understood before Felix Klein (1849-1925) republished the results and explained why Riemann's results were so important.

Riemann

Although only living to age 40 the incredible richness and creativity of his accomplishments in pure mathematics and mathematical physics cannot be overstated. In addition to his eponymous integral he made fundamental contributions to Complex variables, Riemann surfaces, methods of series solutions to differential equations still in use, special functions, number theory, algebraic geometry and the beginning of manifold theory in higher dimensions. This last led to the Klein's Erlangen program and eventually to Einstein's General Relativity.

Riemann

From Riemann's Inaugeral Lecture in support of his Habilitationsshrift.

> *…in a discrete manifold the principle of metric relations is already contained in the concept of the manifold, but in the continuous one it must come from something else. Therefore, either the reality underlying Space must form a discrete manifold, or the basis for the metric relations must be sought outside it, in binding forces acting upon it.*

Erlangen Program

Klein’s 1872 Erlangen Program, classifying geometries by their underlying symmetry groups, was a tour de force, combining many different areas of mathematics with physical application a primary goal.

He had many students who developed these ideas, including Carl Runge, Max Planck, Luigi Bianchi, and Gregorio Ricci-Curbastro (1853-1925).

Ricci-Curbastro was the inventor of Tensor Calculus and the Ricci tensor together with Levi-Civita in 1904.

Tullio Levi-Civita (1873-1929) Technical tools: the Levi-Civita connection

His Swedish colleague Sophus Lie (1842-1899) told Klein about the concept of group and they worked both together and independently on this project.

Hermann Minkowski (1864-1909) was at Göttingen too.

Slightly before Ricci-Curbastro and Levi-Civita, Elwin Bruno Christoffel German mathematician and physicist (1829-1900) studied surfaces. His Christoffel symbols describe a metric connection.

The metric connection is a special type of the affine connection on surfaces or other manifolds endowed with a metric, allowing distances to be measured on that surface.

Many additional concepts follow: parallel transport, covariant derivatives, geodesics, etc.

These concepts can be directly tied to the “shape” of the manifold itself— that shape is determined by how the tangent manifold is related to the cotangent manifold, and this is determined by the metric tensor.

George FitzGerald (1851-1901) in 1889 and Hendrik Antoon Lorentz (1853-1928) in 1892 attempted to explain the result of the Michelson-Morley experiment by proposing a length contraction.

In the meantime French Mathematician Jules Henri Poincaré (1854-1912) was working to emphasize the importance of paying attention to the invariance of laws of physics under different transformations

In his work Poincaré explicitly talked about the “principle of relative motion” (1900) and named it the principle of relativity in 1904, according to which “no physical experiment can discriminate between a state of uniform motion and a state of rest”.

In 1905 Poincaré wrote to Lorentz about Lorentz's paper of 1904, which Poincaré described as a "paper of supreme importance" and in which he discussed what is now called the Lorentz transformation—preserves the space-time interval between events and stabilizes the origin—as distinguished from the Galilean transformations which are generated by time and space translations and spatial uniform motion and a spatial rotation. The Galilean transformations form a 10-dimensional Lie group and are the set of Euclidean isometries.

Poincaré Group: All the Lorentz transformations plus translations. These also form a 10-dimensional Lie group.

The Poincaré Group is exactly the set of Minkowski space isometries.

Amalie (Emmy) Noether (1882-1935) was a student of both David Hilbert and Felix Klein.

Emmy Noether herself was an interesting person. She was a protégé of David Hilbert and Felix Klein at Góttingen and was described by many (including Einstein) as the greatest female mathematician who ever lived.

The Noether Theorems (1915) on the relationship between conserved quantities and symmetry of physical law were regarded by Einstein as beyond profound. She had numerous students who revered her, in spite of her eccentricities, and is widely acknowledged as the “mother” of modern abstract algebra.

Her unfortunate early death at age 53 deprived the world of a great genius.

Apparently (from notes after visit to Prague) in 1912 Einstein realized for the first time that spacetime coordinates need not determine the distances between spacetime points.

Einstein's quote later from his "Autobiographical Notes" :

> *Why were another seven years required for the construction of the general theory of relativity? The main reason lies in the fact that it is not so easy to free oneself from the idea that coordinates must have an immediate metrical meaning*

Our Internal Models and Senses are, At last, Not Enough

Einstein's big contribution was to decide to take geometry seriously and see where that leads—that the (Minkowski) geometrization of physics was not just a calculation trick but an expression of SOMETHING real. Oh yeah, and he improved notation a nontrivial amount.

So we have a blending: reality is still a collection of quantities that are measurable but combined with something that is imaginable by humans only by vague, counter-intuitive, *disturbing* analogy, an abstract object that must be taken seriously because our natural innate internal models of reality, the "meta-world" that gives meaning to our mathematical models, cannot help us.

Why Aren't Senses, our Intuition, Enough?

First: Underdetermination. Many models are consistent with any data.

We must ask: What is the *purpose* of our model-making? Why are we slightly repelled (face it) by QM? Why was it so hard to accept SR in 1905, GR in 1915?

More importantly… Why *should* our model-making machinery match the parts of reality we are exploring now?

What can we do about our limitations?

A partial return to Forms—coupled with experiment.

Trust the models: follow the models where they lead as a substitute for natural intuition. Projection onto a Hilbert subspace after a measurement? Entangled states = no matter of fact regarding the “true state” until a state is measured? Moving clocks run slow, yardsticks shrink in the direction of their motion?

We must pay—at least some—homage to the “pale shadow on the wall of our cave” interpretation of reality.

ARTIFICIAL ASSISTANCE

Game of Life with Bells and Whistles

Natural Evolution, within a simulation, with individuals challenged by QM or GR for survival

Run the game until intelligence evolves, for which QM or GR will be instinctive, natural

And…what about us? Are we, with our Newtonian instincts, helping "something" understand the patterns we have evolved to see?

Theseus Ponders His Creator

The Poet's Eye in a Fine Frenzy Rolling

Doth Glance From Heaven to Earth, From Earth to Heaven,

And as Imagination Bodies Forth the Forms of Things Unknown

The Poet's Pen Turns Them to Shapes, and Gives to Airy Nothing

A Local Habitation and a Name.

William Shakespeare,
A Midsummer Night's Dream