THE MONTY HALL PROBLEM

This is a classic introductory probability problem with counter-intuitive aspects based on a popular TV game show.

In this show game host Monty Hall picks a contestant who selects one of three curtains on a stage. Behind one curtain is a valuable prize while two conceal junk.

Monty knows where the good stuff is, and there is at least one unselected curtain that conceals junk. Monty opens a junk curtain and then asks the contestant if he or she wishes to stick with the initial "intuition" or switch to the remaining unopened curtain.

The question here is how should the contestant proceed, and why.

One approach is to simply state that the only way the remaining unopened curtain is NOT the "good curtain" is if the contestant guessed right to begin with. There was one chance in three of *that* so there is two-thirds chance of being a winner if you switch.

Beginning students often find this unconvincing, but there are other approaches in terms of sample spaces (which these students just learned about) that could help.

The original picking corresponds to a very simple 9 element sample space and

$$\{\, (m,n) \ |\ m,n = 1,2,3 \,\} \qquad P(m,n) = 1/9 \text{ for all } m,n.$$

In this sample space (m,n) corresponds to the good stuff behind curtain m and the contestant picks curtain n. The probability is (presumably) equal for each of these 9 combinations.

So the event "contestant guessed right" is

$$\{\, (1,1),\ (2,2),\ (3,3) \,\}$$

which has probability $1/9 + 1/9 + 1/9 = 1/3$.

Adding the additional stage to the game changes the sample space. Monty selects the curtain he opens based on his additional knowledge.

If the contestant guessed wrong to begin with then Monty has no choice: he must pick the single junk curtain that remains. But if the contestant guessed right then Monty must decide which of the two possible junk curtains he will open.

We will presume that if Monty has a choice of two curtains to open he picks one of the two randomly. (In fact, we will see that *how* he makes his choice in this case is not relevant to the solution.)

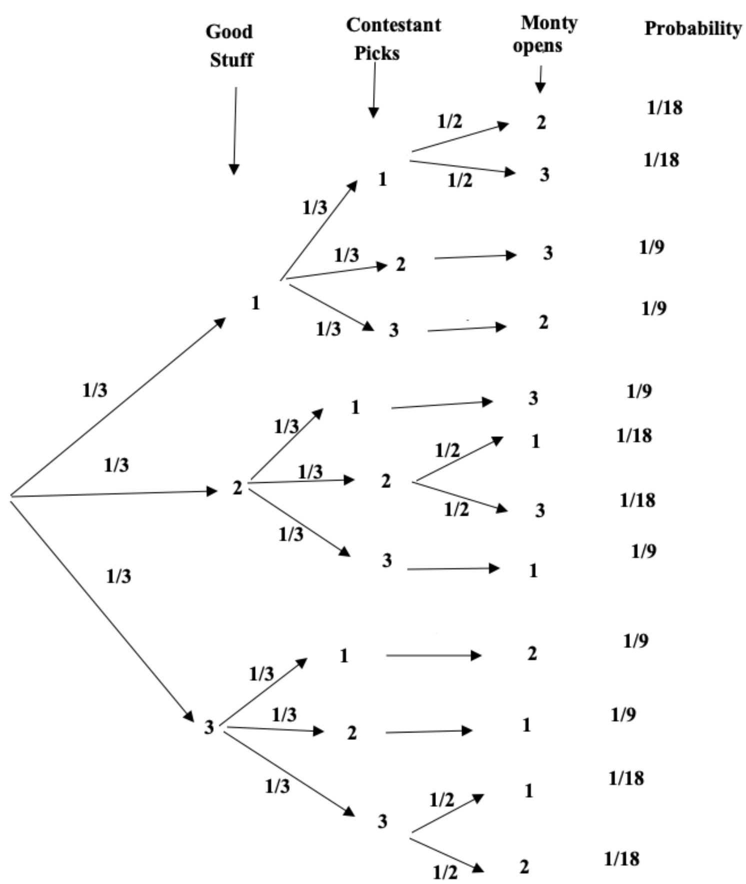
We now have twelve possible elementary outcomes instead of nine, and some are more likely than others.

112: Good stuff behind 1, contestant picks 1, Monty opens 2
113: Good stuff behind 1, contestant picks 1, Monty opens 3
123: Good stuff behind 1, contestant picks 2, Monty opens his only choice 3

132: Good stuff behind 1, contestant picks 3, Monty opens his only choice 2
213: Good stuff behind 2, contestant picks 1, Monty opens his only choice 3
221: Good stuff behind 2, contestant picks 2, Monty opens 1
223: Good stuff behind 2, contestant picks 2, Monty opens 3
231: Good stuff behind 2, contestant picks 3, Monty opens his only choice 1
312: Good stuff behind 3, contestant picks 1, Monty opens his only choice 2
321: Good stuff behind 3, contestant picks 2, Monty opens his only choice 1
331: Good stuff behind 3, contestant picks 3, Monty opens 1
332: Good stuff behind 3, contestant picks 3, Monty opens 2

The six outcomes where Monty has but a single choice of curtain to open have probability 1/9. But the six outcomes for which Monty has two choices have probaility 1/18 each.

We use a tree diagram to illustrate and keep track of the situation below.



The event "the contestant switches and wins" is

$$W = \{\, 123,\ 132,\ 213,\ 231,\ 312,\ 321 \,\}$$

and each of these elementary outcomes has probability 1/9.

So $P(W) = 6/9 = 2/3$.