DISCUSSION OF CHAPRA'S PROBLEM 23.15 WITHOUT THE SMALL-ANGLE APPROXIMATION TO THE DE

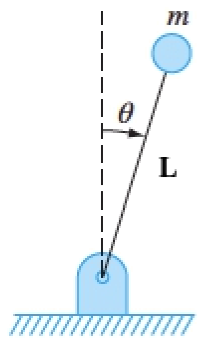

FIGURE P23.15

Initial conditions to be specified are: $g,\ L,\ m,\ \theta_0 = \theta(0),\ \dot{\theta}_0 = \dot{\theta}(0)$.

Place the origin at the bearing so $x = L * \sin(\theta)$ and $y = L * \cos(\theta)$.

The motion is confined to the circle so the magnitude of velocity, v, is $L * \dot{\theta}$.

We assume a rigid rod, no friction at the bearing, no air resistance and the assembly is mounted on the edge of a table so that full rotations are possible.

Conservation of energy provides a constant of the motion:

$$E = m * g * y + \frac{1}{2} * m * v^2 = m * g * L * \cos(\theta) + \frac{1}{2} * m * L^2 * \dot{\theta}^2$$

Differentiating with respect to time produces

$$0 = -\, m * g * L * \sin(\theta) * \dot{\theta} + m * L^2 * \dot{\theta} * \ddot{\theta}$$

Assuming $\dot{\theta}$ to be nonzero, cancel it (and m and L) from the equation to produce the DE of the system

$$\ddot{\theta} = \frac{g}{L} * \sin(\theta).$$

Chapra's equation $\ddot{\theta} = \frac{g}{L} * \theta$ is a good approximation for smallish, but not necessarily tiny, angles θ.

Validity for small θ rests on the fact (using the "big oh" notation) that

$$\sin(\theta) = \theta + \mathcal{O}(\theta^3).$$

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