Quantum Mechanics Preview

History

Planck $\rightarrow$ black box radiation $\rightarrow$ quantization

Heizenberg $\rightarrow$ matrix mechanics $\rightarrow$ calculation

von Neumann $\rightarrow$ mathematical foundations

Definitions/Assumptions

- "states" of a physical system $\approx$ certain members of an infinite dimensional vector space, $\mathcal{H}$, with notions of magnitude, distance & angle

  $\approx$ functions $\{\varphi_t(x)\}$ [x just a dummy variable; t time]

  * $\varphi_t(x)$ embodies all knowable information about the system at time t

- "observables" of a system $\approx$ certain linear transformations on the "space of states"

  $\approx \{A : \mathcal{H} \rightarrow \mathcal{H}\}$ [eigenvalues & eigenvectors $A(\alpha_t(x)) = a_n \cdot \alpha_t(x)$]

- to measure observable A when system is in state $\varphi_t(x)$ $\approx$ evaluate A at $\varphi_t(x)$ [$A(\varphi_t(x)) = ?$]

  * measurement of observable A can only yield one of the eigenvalues a_n.

Tenets of quantum physics

- "Small-scale" universe is probabilistic; i.e., strongest predictive statement about measurement of a physical system is necessarily probabilistic.

  [observable A measured when system in state $\varphi_t \Rightarrow prob(a_n) = \left|(\alpha_t, \varphi_t)\right|^2$]
- Measurement alters the state of the system, uncontrollably. [$A(\varphi_t(x)) = \psi_t(x)$]
- Two observables, A and B, may not be simultaneously measurable. [Is $A \circ B = B \circ A$?]

Examples of linear transformations "on functions"

$D(f(x)) = f'(x) = \frac{df}{dx}$ $\qquad$ $M(f(x)) = x \cdot f(x)$

Problems

1. Demonstrate that D and M do not commute; i.e. $D \circ M \neq M \circ D$.
2. For which functions, $f(x)$, is it true that $D(M(f)) = M(D(f))$?
3. For which functions, $f(x)$, is it true that $D(M^2(f)) = M^2(D(f))$?
4. The matrix $S = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ defines a linear transformation from R^3 to R^3.

   a) Calculate S^2.

   b) Show that if $A \circ S = S \circ A$, then $A = q(S)$, where $q(x)$ is a quadratic polynomial.