Hilbert Space Methods Used in a First Course in Quantum Mechanics

Quantum Mechanics Preview

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History

Planck Planck $\Longrightarrow$ Black Box Radiation Planck $\Longrightarrow$ Black Box Radiation $\Longrightarrow$ Quantization

Heizenberg Heizenberg $\Longrightarrow$ Matrix Mechanics Heizenberg $\Longrightarrow$ Matrix Mechanics $\Longrightarrow$ Calculation

von Neumann von Neumann $\Longrightarrow$ Mathematical Foundations

States

States of a physical system

$\Longleftrightarrow$

certain members of an infinite dimensional vector space $\mathfrak{H}$ with notions of distance and angle

$\Longleftrightarrow$

functions $\{ \varphi_t(x) \}$ [x just a dummy variable; t time].

Observables

Observables of a physical system

$$\Longleftrightarrow$$

certain linear transformations on the *space of states*

$$\Longleftrightarrow$$

$$\{\, \mathcal{A} \colon \mathfrak{H} \to \mathfrak{H} \,\}$$

[eigenvectors and eigenvalues: $\mathcal{A}(\alpha_t(x)) = a_n \alpha_t(x)$].

Measurement

Measurement of observable $\mathcal{A}$ when a system is in state φ_t

$$\Longleftrightarrow$$

Evaluate $\mathcal{A}$ at φ_t $[\mathcal{A}(\varphi_t(x)) = ?]$.

$$\Longleftrightarrow$$

Measurement of the observable
can only yield one of the eigenvalues a_n.

Small-scale universe is probabilistic; i.e., strongest predictive statement about measurement of a physical system is necessarily probabilistic.

Observable $\mathcal{A}$ measured when system in state $\varphi_t(x)$:

$$prob(a_n) = |\langle \alpha_t, \varphi_t \rangle|^2 \,.$$

Measurement alters the state of a system uncontrollably.

$$[\mathcal{A}(\varphi_t(x)) = \Psi_t(x)].$$

Two observables $\mathcal{A}$ and $\mathcal{B}$
may not be simultaneously measurable.

[Is $\mathcal{A} \circ \mathcal{B} = \mathcal{B} \circ \mathcal{A}$?]

$$D(f(x)) = f'(x) = \frac{df}{dx} \qquad M(f(x)) = xf(x).$$

- Demonstrate that D and M do not commute; i.e.

$$D \circ M \neq M \circ D.$$

- For which functions $f(x)$ is it true that

$$D(M(f)) = M(D(f))?$$