Seminar Practice Problems
"If I can NOT DO then I don't understand. If I can DO then I might understand."

Some Metrics in R^2 (and easily extendable to R^n)

A (bird metric): $d((x,y),(u,v)) = \sqrt{(x-u)^2+(y-v)^2}$

B (hor-vrt metric): $d((x,y),(u,v)) = |x-u|+|y-v|$

C (max metric): $d((x,y),(u,v)) = \max\{|x-u|,|y-v|\}$

D (discrete metric): $d(A,B) = \{1 \ \text{if} \ A \neq B, 0 \ \text{if} \ A = B\}$ A, B points in R^2

E ("crazy" metric): $d((x,y),(u,v)) = \begin{cases} \sqrt{(x-u)^2+(y-v)^2} & \text{if } \frac{y}{x} = \frac{v}{u} \\ \sqrt{(x^2+y^2)+(u^2+v^2)} & \text{if } \frac{y}{x} \neq \frac{v}{u} \end{cases}$

Open ball with center (a,b) and radius r: **B((a,b),r)** = $\{(x,y) : d((a,b),(x,y)) < r\}$

A Problems: Bird $d((x,y),(u,v)) = \sqrt{(x-u)^2+(y-v)^2}$

A=(3,4), B=(1,5), C=(1,5), D=(-1,2)

1) d(A,B)=_____ d(A,C)=_____

d(B,C)=_____ d(A,D)=_____

2) Sketch the open ball B(((3,4), 2)

B Problems: Hor-Vrt $d((x,y),(u,v)) = |x-u|+|y-v|$

A=(3,4), B=(1,5), C=(1,5), D=(-1,2)

1) d(A,B)=_____ d(A,C)=_____

d(B,C)=___ d(A,D)=_____

2) Sketch the open ball B(((3,4), 2)

C Problems: Max $d((x,y),(u,v)) = \max\{|x-u|,|y-v|\}$

A=(3,4), B=(1,5), C=(1,5), D=(-1,2)

1) d(A,B)=_____ d(A,C)=_____

d(B,C)=____ d(A,D)=_____

2) Sketch the open ball B(((3,4), 2)

D Problems: Discrete $d(A,B) = \{1 \ \text{if} \ A \neq B, 0 \ \text{if} \ A = B\}$

A=(3,4), B=(1,5), C=(1,5), D=(-1,2)

1) d(A,B)=_____ d(A,C)=_____

d(B,C)=____ d(A,D)=_____

2) Sketch the open ball B(((3,4), 0.7), B(((3,4), 2)

E Problems: Crazy $d((x,y),(u,v)) = \begin{cases} \sqrt{(x-u)^2 + (y-v)^2} & \text{if} \ \frac{y}{x} = \frac{v}{v} \\ \sqrt{(x^2+y^2)+(u^2+v^2)} & \text{if} \ \frac{y}{x} \neq \frac{v}{v} \end{cases}$

A=(3,4), B=(1,5), C=(1,5), D=(-1,2)

1) d(A,B)=_____ d(A,C)=_____

d(B,C)=____ d(A,D)=_____

2) Sketch the open ball B(((3,4), 2)

3) Sketch the open ball B(((3,4), 6)

Sequence Space

Points in Sequence Space:

$P = sequence\{p_n\} = \{p_1, p_2, p_3, \ldots\}, \quad Q = sequence\{q_n\} = \{q_1, q_2, q_3, \ldots\}$

Metrics for Sequences:

$l_2 \ \ metric: \ d(P,Q) = \sqrt{\sum_{n=0}^{\infty} (p_n - q_n)^2}$

Abs Value metric: $d(P,Q) = \sum_{n=0}^{\infty} | p_n - q_n |$

Sup metric: $d(P,Q) = \sup\{| p_n - q_n | \ for \ \ all \ \ n\}$

Example Points: $A = sequence\{a_n: \ a_0 = 0, \ \ a_n = \frac{1}{n^2} \ \ if \ \ n > 0\}$,

$B = sequence\{b_n: \ b_0 = 0, \quad b_n = \frac{1}{n^3} \ \ if \ \ n > 0\}$,

Z=seq$(z_n = 0 \ \ for \ all \ \ n)$

$C1 = sequence\{c_n: \ \ c_n = 1 \ \ if \ \ n = 1, \ c_n = 0 \ \ if \ \ n \neq 1\}$

$C2 = sequence\{c_n: \ \ c_n = 1 \ \ if \ \ n = 2, \ c_n = 0 \ \ if \ \ n \neq 2\}$

Problems:

$l_2 \ \ metric$: d(A,B)=

d(A,C1)=

d(C1,C2)=

Is A in B(z, 1)? Is C1 in B(A, 1)? Is C1 in B(C2, 1.5)?

Abs Value metric: d(A,B)=

d(A,C1)=

d(C1,C2)=

Is A in B(z, 1)? Is C1 in B(A, 1)? Is C1 in B(C2, 1.5)?

Sup metric: d(A,B)=

d(A,C1)=

d(C1,C2)=

Is A in B(z, 1)? Is C1 in B(A, 1)? Is C1 in B(C2, 1.5)?

A Function Space

One example: C[0,1] ={ f: is continuous for 0<=x<=1}

Example Points in C[0,1]: $f(x) = x,\ g(x) = x^2,\ h(x) = 1$, z(x)=0,

$p_n(x) = (n(1 - nx)\ \ if\ \ 0 \le x \le 1/n,\ 0\ \ if\ \ 1/n < x)$

Metrics for C[0,1]:

L2: $d(f,g) = \sqrt{\int_0^1 \big(f(x) - g(x)\big)^2\ dx}$

Abs value: $d(f,g) = \int_0^1\ |\, f(x) - g(x)\, |\ dx$

Sup: $d(f,g) = \sup\{\ |\ f(x) - g(x)\ |\ for\ \ x \in [0,1]\ \}$

A Problems: using L2 metric

d(f,z) = _____ d(g,z) = _____ d(p_3,z) = _____

Is f in B(z, 1)? Is g in B(z, 1)? Is p_n in B(z, 1.2)?

B Problems: using Abs Val metric

d(f,z) = _____ d(g,z) = _____ d(p_3,z) = _____

Is f in B(z, 1)? Is g in B(z, 1)? Is p_n in B(z, 1.2)?

C Problems: using Sup metric

d(f,z) = _____ d(g,z) = _____ d(p_3,z) = _____

Is f in B(z, 1)? Is g in B(z, 1)? Is p_n in B(z, 1.2)?