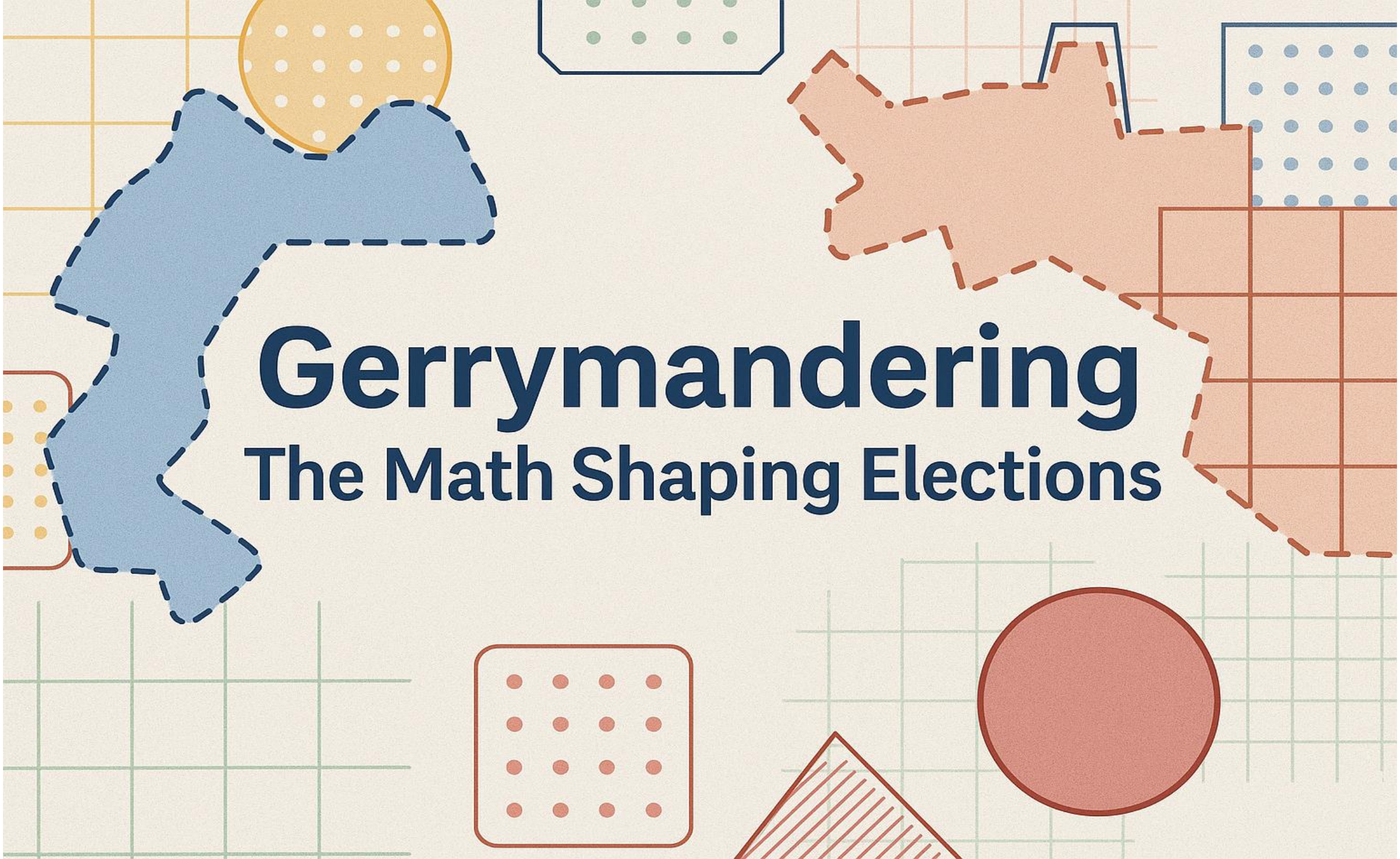
Gerrymandering
The Math Shaping Elections

US Federal Legislative

- Senate:
  - 100 Members (2/state)
- House of Representatives
  - 435 Members
  - Number depends on population
  - Within a State, a geographic regions (districts) are each responsible for electing one representative

Redistricting: redrawing these districts

Gerrymandering

- Drawing district lines “to accomplish a sinister or unlawful purpose”

- Historically 3 main reasons why:
  - To benefit/harm a racial group
  - To benefit/harm a political party
  - To benefit/harm an incumbent

How much power do the map-drawers have?

Purple has 60% of the total vote, Yellow has 40%

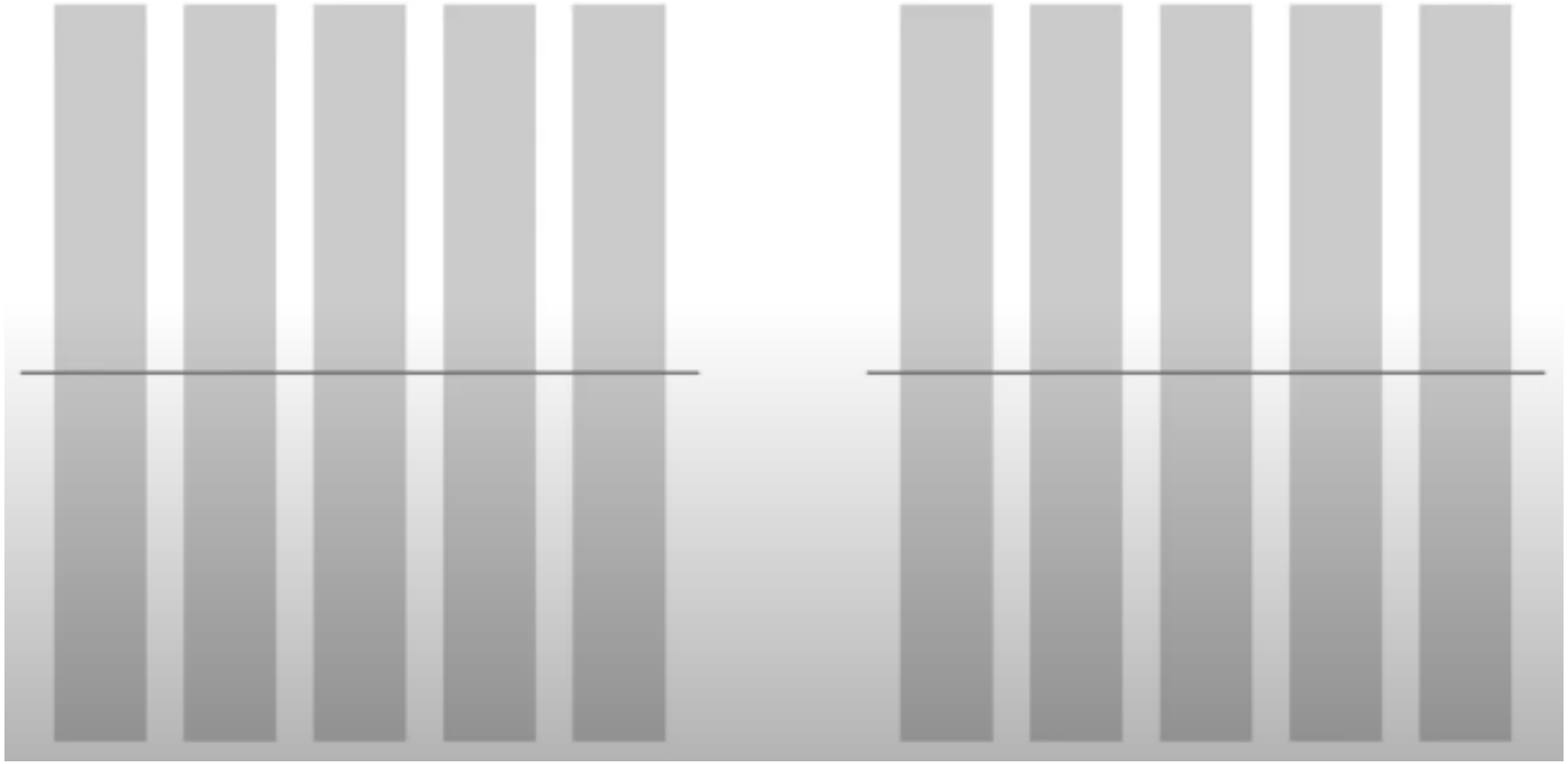

Image from Moon Duchin's 2018 Talk on The Math of Redistricting

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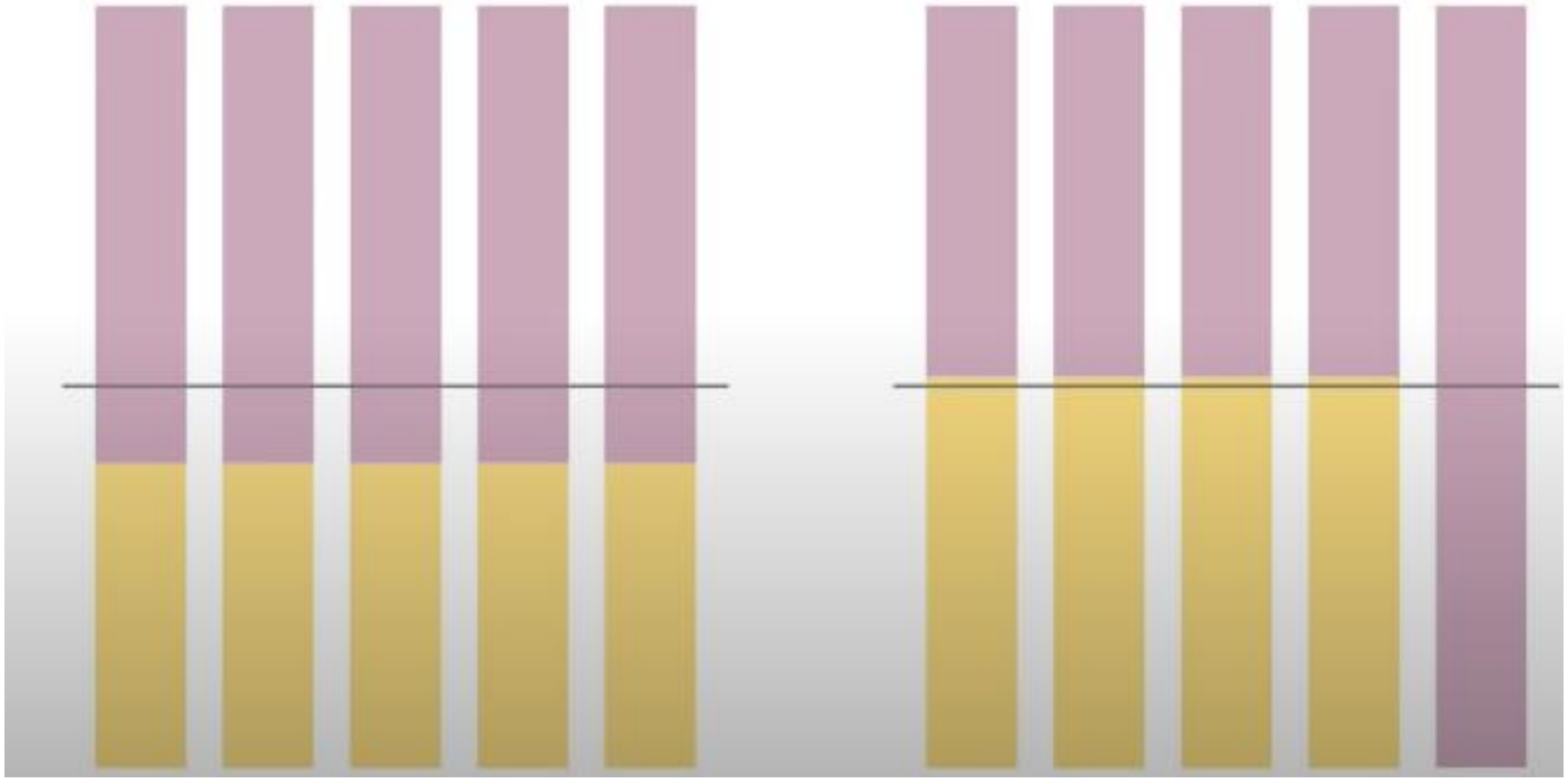


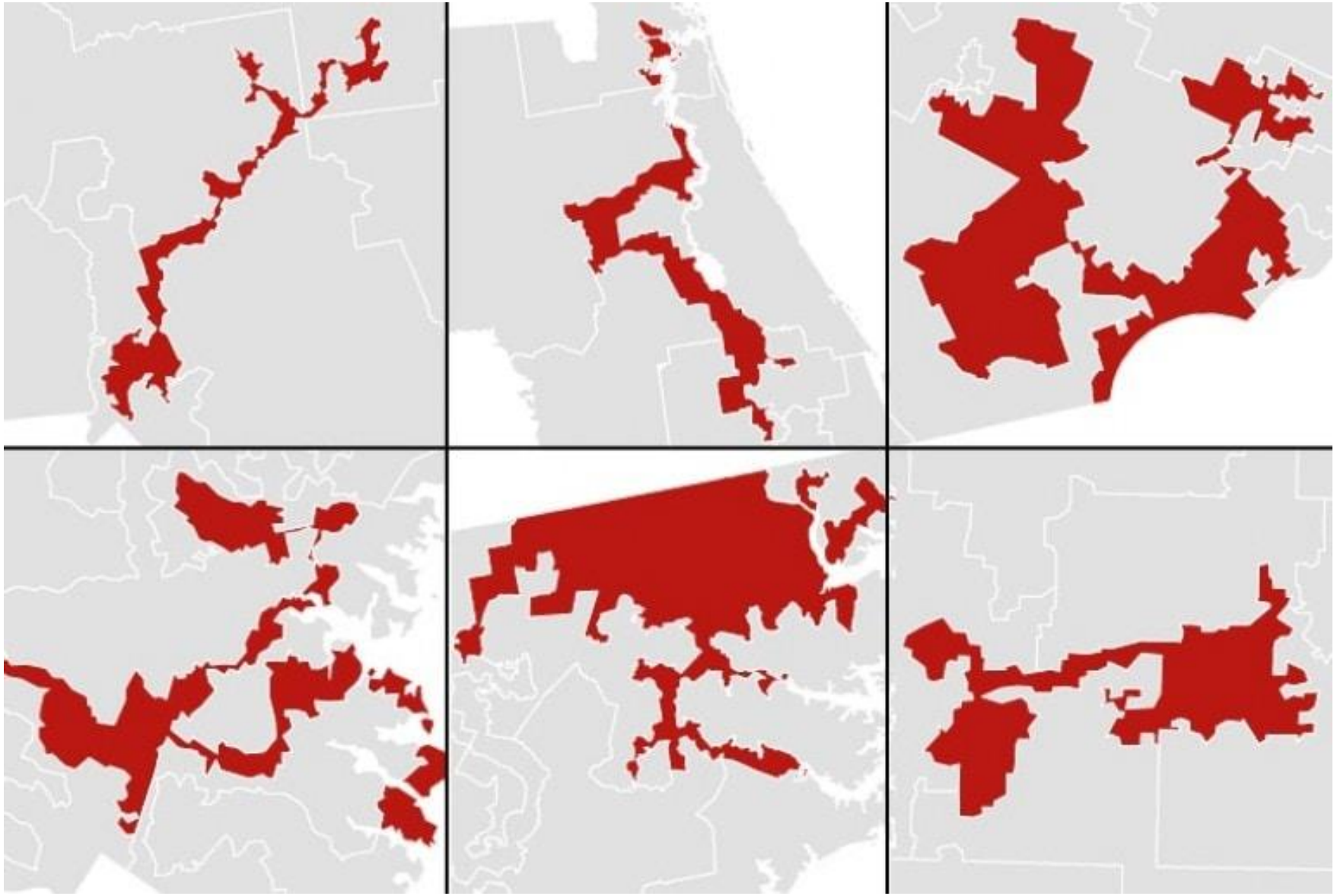
Image from Moon Duchin's 2018 Talk on The Math of Redistricting


METHUEN
HAVERHILL
AMESBURY
SALSBURY
BRADFORD
N.PORT
NEWBURY
ANDOVER
ROXFORD
ROWLEY
MIDDLETON
TOPSFIELD
IPSWICH
LYNNFIELD
DANVERS
HAMILTON
WENHAM
BEVERLY
MANCHESTER
GLOCESTER
SALEM
MARBLE HEAD
LYNN
CHELSEA

NC-12

FL-5

PA-7



MD-3

NC-1

TX-33

Math for the People
Mark Branson,
Whitney George

“Ground Rules” for districting

- Each district elects one representative
- Districts should have equal population
- Districts should be “as compact as possible”
- Districts should be a single connected piece
- Respect county and city boundaries
- Respect “communities of interest”
- Voting Rights Act (1965)
  - Guarantees “opportunity to elect a candidate of choice”

What do we *want* from a districting plan?

What do we *want* from a districting plan?

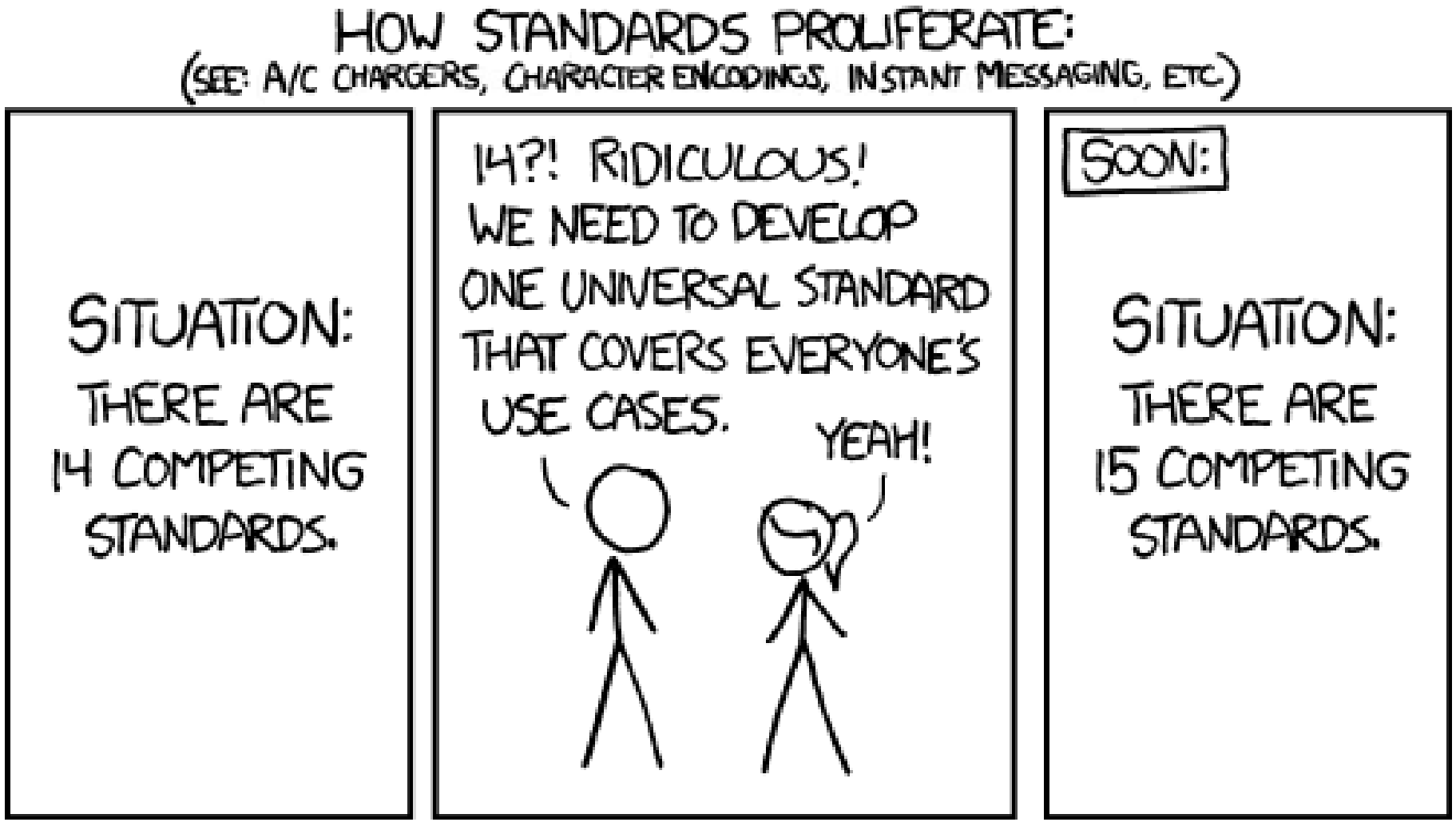
- Voice for Minority
- Voice for Majority
- Proportionality (“fairness”)
- Competitiveness
- Stability
- Regular Shapes
- Representatives that speak for the population

Compactness

Compactness

A = Area of District
P = Perimeter of District

• Square Reock	A/(area of minimum bounding square)
• Circle Reock	A/(area of minimum bounding circle)
• Convex Hull	A/(area of convex hull)
• Polsby-Popper (circle)	$4\pi A/P^2$
• Polsby-Popper (square)	$16A/P^2$
• X-y skew	Width/Length
• Moment of Inertia	Average squared distance of area from the district's centroid
• Population Dispersion	Average distance of people from the district's centroid


HOW STANDARDS PROLIFERATE:
(SEE: A/C CHARGERS, CHARACTER ENCODINGS, INSTANT MESSAGING, ETC.)
SITUATION: THERE ARE 14 COMPETING STANDARDS.
14?! RIDICULOUS! WE NEED TO DEVELOP ONE UNIVERSAL STANDARD THAT COVERS EVERYONE'S USE CASES.
YEAH!
SOON:
SITUATION: THERE ARE 15 COMPETING STANDARDS.

Xkcd.com/927

What is fair?

2024 United States House of Representatives elections in Washington

← 2022 **November 5, 2024** 2026 →

All 10 Washington seats to the United States House of Representatives

Party	Democratic	Republican
Last election	**8**	2
Seats won	**8**	2
Seat change	–	–
Popular vote	**2,155,907**	1,592,599
Percentage	**57.21%**	42.26%
Swing	▼ 0.67%	▲ 0.56%

Gerrymandered?

What is fair?

55 A 45 B	55 A 45 B	55 A 45 B
55 A 45 B	55 A 45 B	55 A 45 B
55 A 45 B	55 A 45 B	55 A 45 B

495 votes for party A (55%)

405 votes for party B (45%)

What is fair?

55 A 45 B	55 A 45 B	55 A 45 B
55 A 45 B	55 A 45 B	55 A 45 B
55 A 45 B	55 A 45 B	55 A 45 B

495 votes for party A (55%)

405 votes for party B (45%)

A wins 3/3

What is fair?

55 A 45 B	55 A 45 B	55 A 45 B
55 A 45 B	55 A 45 B	55 A 45 B
55 A 45 B	55 A 45 B	5 A 95 B

445 votes for party A (49.5%)

455 votes for party B (50.5%)

What is fair?

55 A 45 B	55 A 45 B	55 A 45 B
55 A 45 B	55 A 45 B	55 A 45 B
55 A 45 B	55 A 45 B	5 A 95 B

445 votes for party A (49.5%)

455 votes for party B (50.5%)

A wins 2/3

What is ~~fair?~~ Not gerrymandered?

Measures of an advantage

- Efficiency Gap (EG): Difference in number of wasted votes/ total votes
- Partisan Bias (PB): The difference in seat share each party would receive if both parties earned 50% of the vote
- Mean-Median (MM): What percentage of the vote a party needs in order to win 50% of the vote

Compare suspected map to a “typical” map

What is ~~fair?~~ Not gerrymandered?

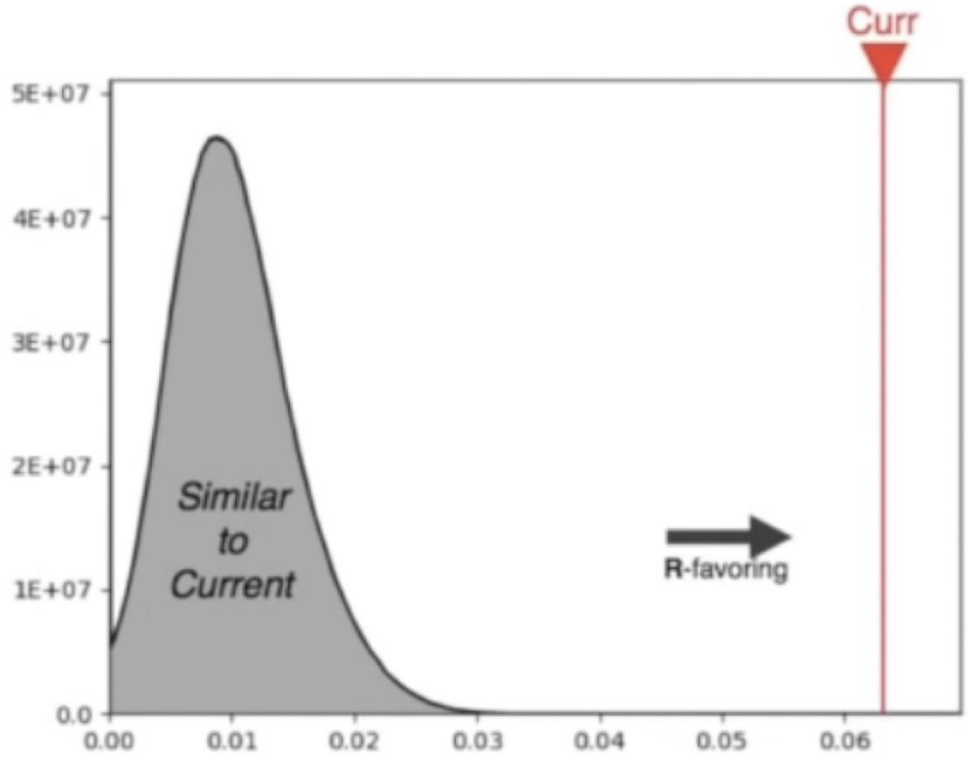
Measures of an advantage

- Efficiency Gap (EG): Difference in number of wasted votes/ total votes
- Partisan Bias (PB): The difference in seat share each party would receive if both parties earned 50% of the vote
- Mean-Median (MM): What percentage of the vote a party needs in order to win 50% of the vote

Compare suspected map to ~~a~~ Millions of "typical" map

What is ~~fair?~~ Not gerrymandered?

Assume no gerrymandering, what are the odds of generating a map this biased just due to random chance?

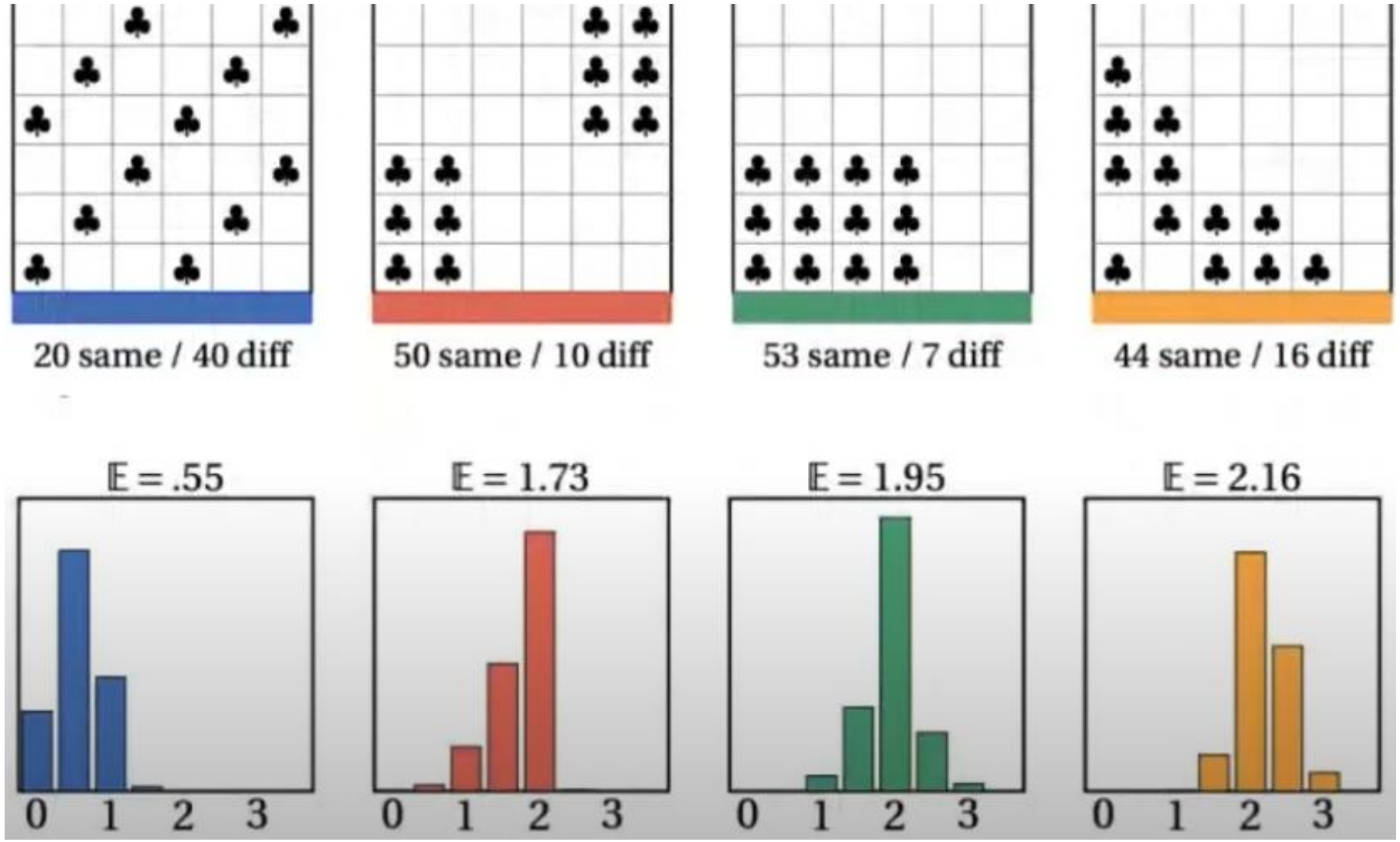


2018 PA Mean-Median Scores

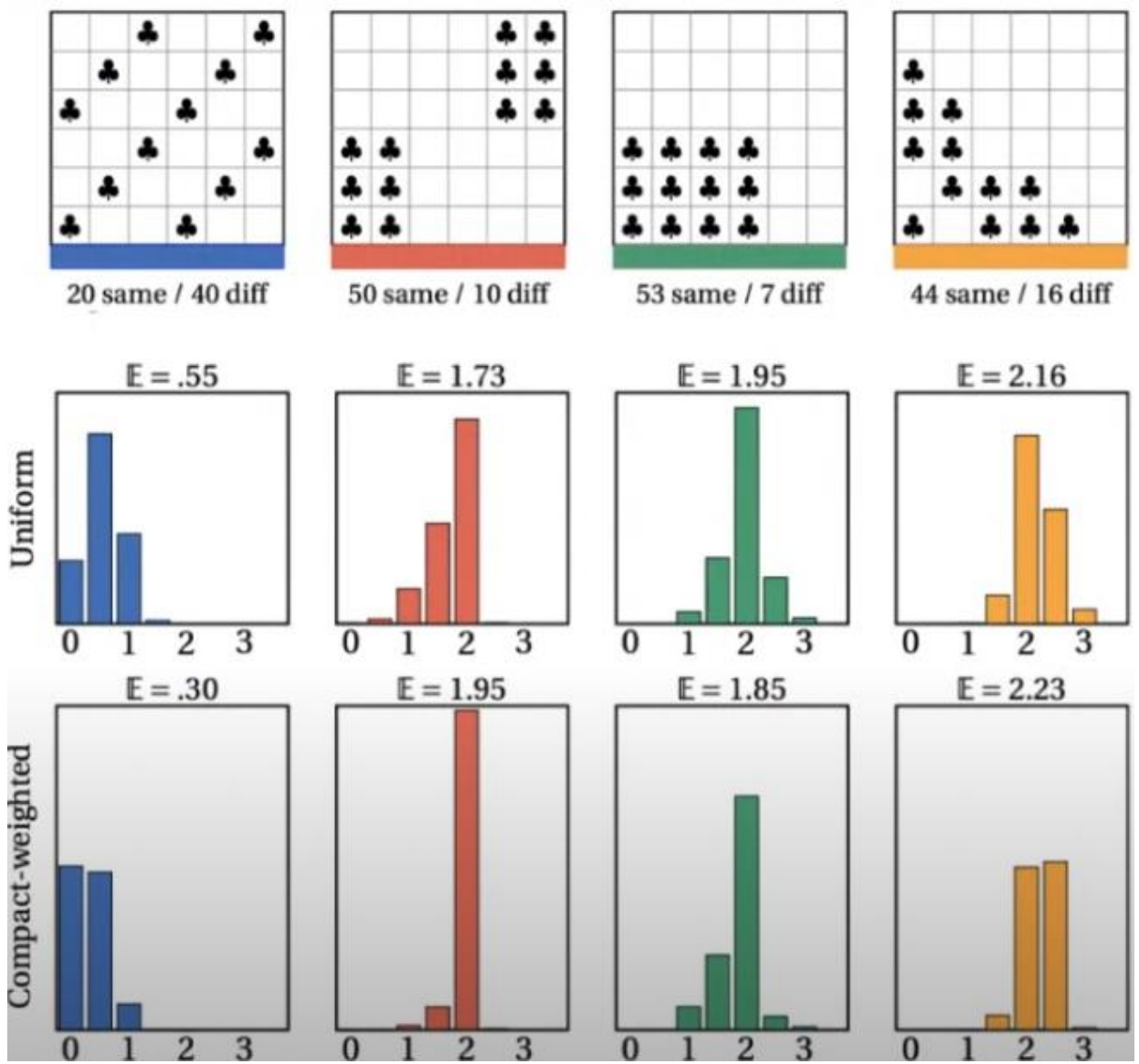
How many maps are possible?

	Rook					Queen				
	equal size	± 1	± 2	± 3	all, non-∅	equal size	± 1	± 2	± 3	all, non-∅
2x2 → 2	2	6	6	6	6	3	7	7	7	7
3x3 → 3	10	58	170	226	258	36	224	622	746	782
4x4 → 2	70	206	338	454	627	1,718	4,690	6,630	7,598	8,171
4x4 → 4	117	1,953	8,849	29,069	62,741	2,620	46,456	222,398	710,622	1,130,612
4x4 → 8	36	34,444	153,094	251,230	340,198	280	567,725	2,629,081	4,262,917	5,362,109
5x5 → 5	4,006	193,128	1,472,750	6,194,822	72,137,699	1,397,790	70,491,206	565,708,110	2,467,619,460	19,258,645,522
6x6 → 2	80,518	241,218	400,727	557,031	1,123,743	?	?	?	?	2,668,525,423
6x6 → 3	264,500	1,848,872	5,006,968	9,714,644	99,699,033	?	?	?	?	981,728,132,977
6x6 → 4	442,791	8,313,719	36,716,529	98,715,441	2,872,166,677	?	?	?	?	33,178,431,625,979
6x6 → 6	451,206	61,412,106	787,941,042	4,564,190,094	356,612,826,084	6,025,727,349	?	?	?	3,628,666,881,233,030
6x6 → 9	178,939	600,896,399	44,824,927,867	1,246,776,785,419	27,173,118,202,682	1,411,875,619	?	?	?	193,647,162,942,941,072
6x6 → 12	80,092	4,555,902,726	2,994,170,540,354	19,290,275,421,883	217,674,971,893,169	245,410,623	?	?	?	1,045,944,428,847,211,954
6x6 → 18	6,728	321,276,916,270	13,415,885,335,894	49,804,910,882,006	146,904,919,330,547	3,037,561	?	?	?	240,051,309,075,264,719
7x7 → 7	158,753,814	?	?	?	7,146,137,621,219,723	?	?	?	?	7,291,155,679,611,862,880,190
8x8 → 8	187,497,290,034	?	?	?	556,983,247,769,141,192,388	?	?	?	?	?
9x9 → 9	706,152,947,468,301	?	?	?	163,738,881,245,258,156,011,991,945	?	?	?	?	?

MGGG Redistricting Lab: mggg.org



From Moon Duchin's talk @ Georgia Tech, 2021



From Moon Duchin's talk @ Georgia Tech, 2021

Links to Check out

- Metric Geometry and Gerrymandering Group https://mggg.org/
- Princeton Gerrymandering Project: https://gerrymander.princeton.edu/
- Dave's Redistricting: https://davesredistricting.org/
- Exploring Seat-Vote Curves:
  - https://jeffreyshen19.github.io/Seats-Votes-Curves/