POTENTIAL ENERGY DIAGRAMS & PROBABILITY DENSITY

Classical ideas that are useful in quantum mechanics

Motivation
Teaching PHYS 225 (Quantum Physics) this quarter
Student understanding of these topics was focus of my dissertation

About me

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Undergrad & grad at UW Seattle

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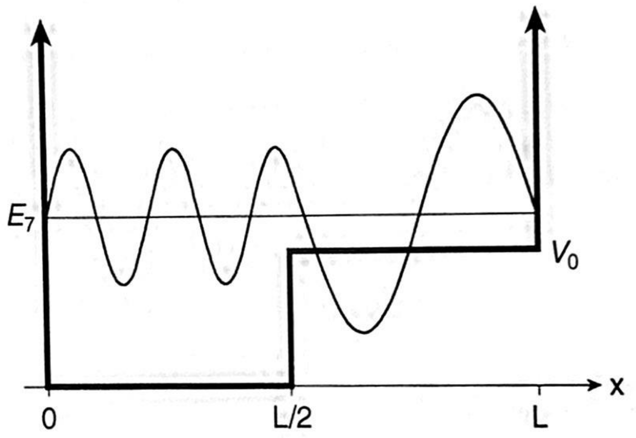

Classical potential energy diagrams → Classical probability density → Quantum mechanics

What are they?
How should we think of them?
What is difficult about learning them?

How can our classical knowledge help us?

Diagram I want my PHYS 225 students to understand

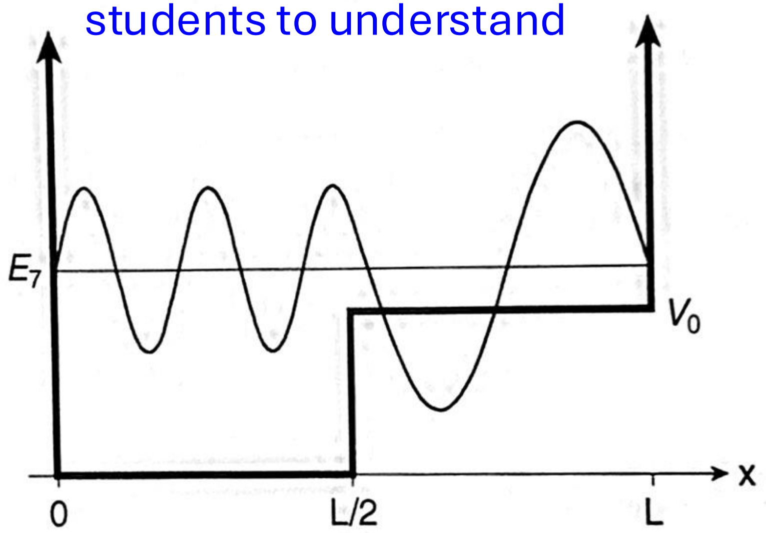

FIGURE 5.27 An energy eigenstate of the asymmetric square well.

Classical potential energy diagrams → Classical probability density → Quantum mechanics

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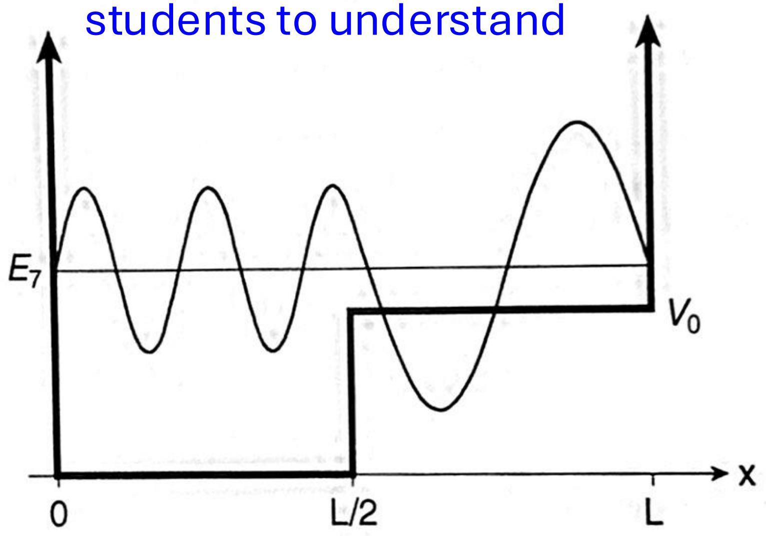
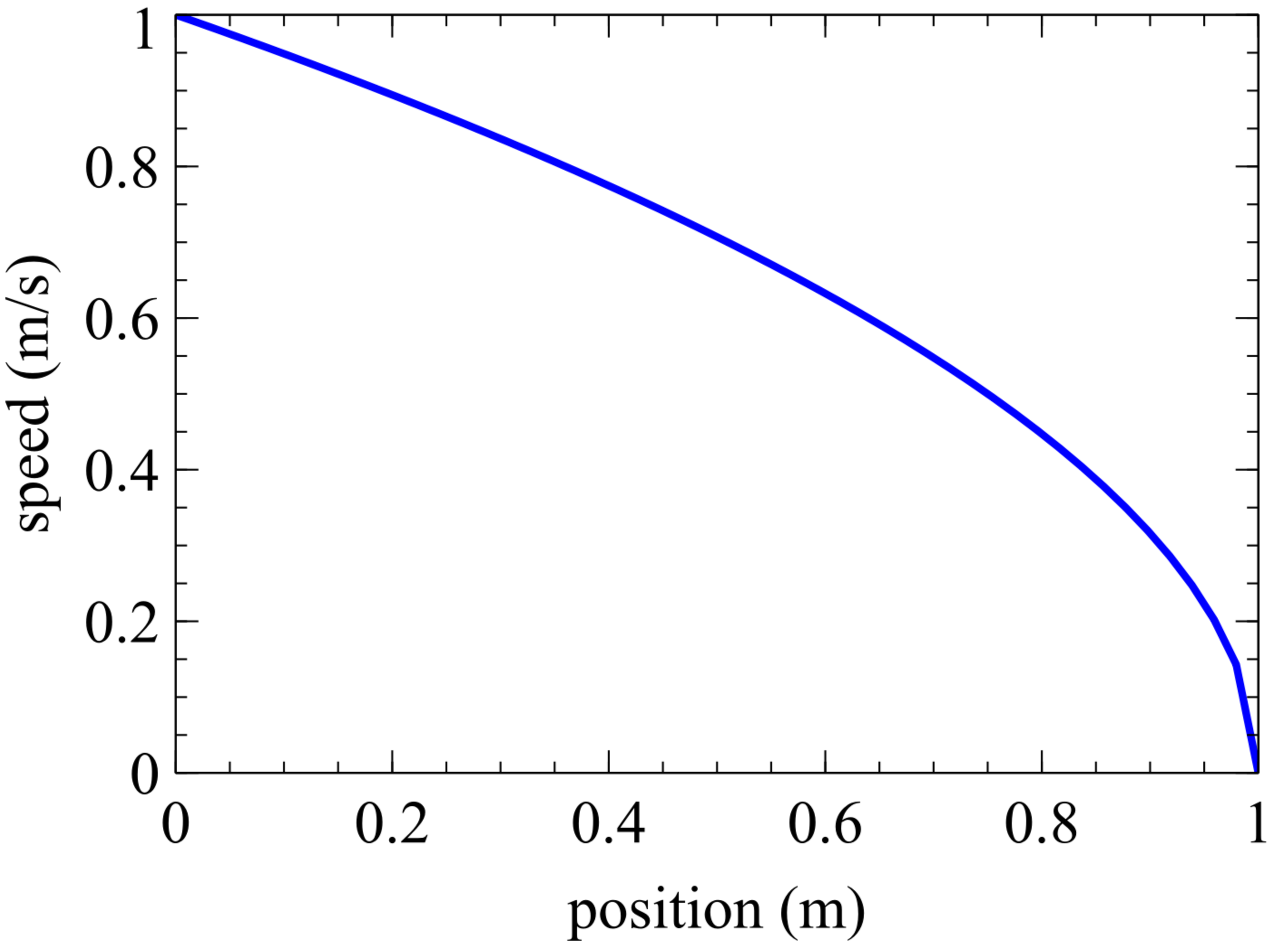


FIGURE 5.27 An energy eigenstate of the asymmetric square well.

A question we'll come back to

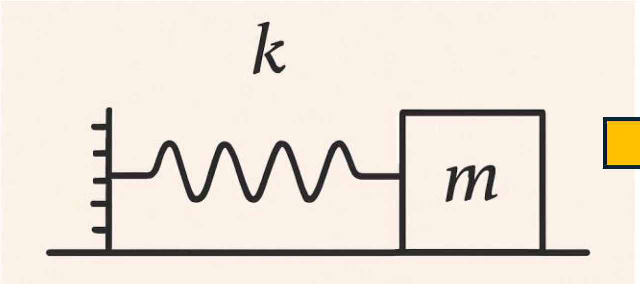


Think about. Will revisit.

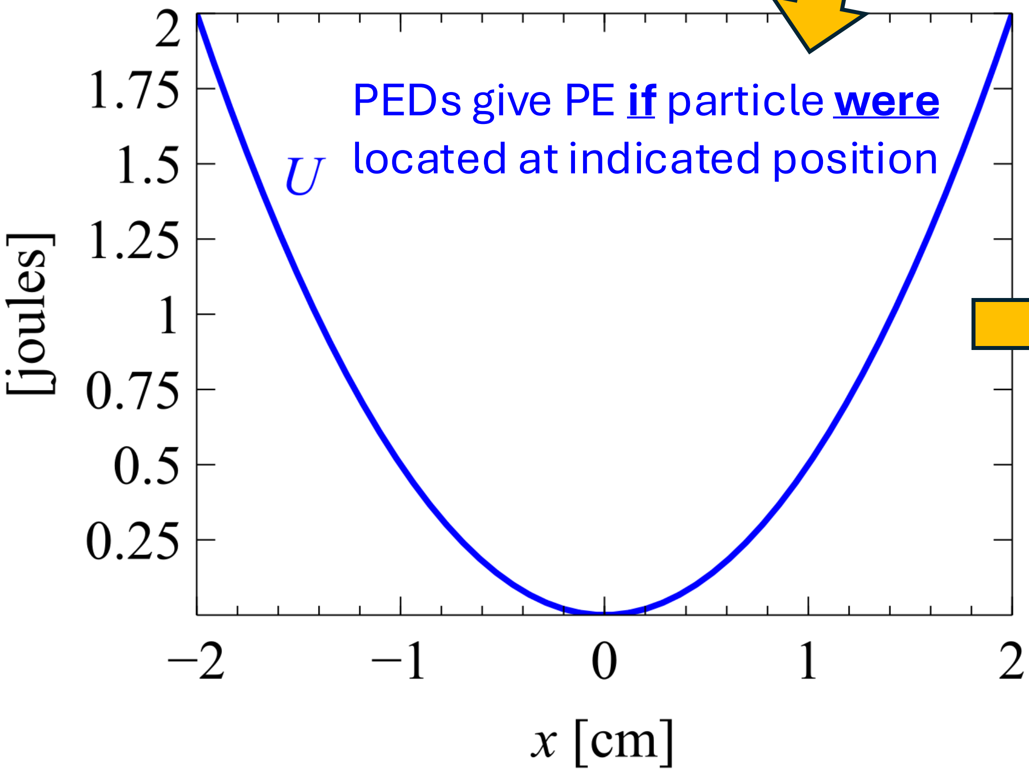
When the particle is at 0.4 m, which best describes its motion?

- *Speeding up*
- *Slowing down*
- *Either speeding up or slowing down*
- *Constant speed*

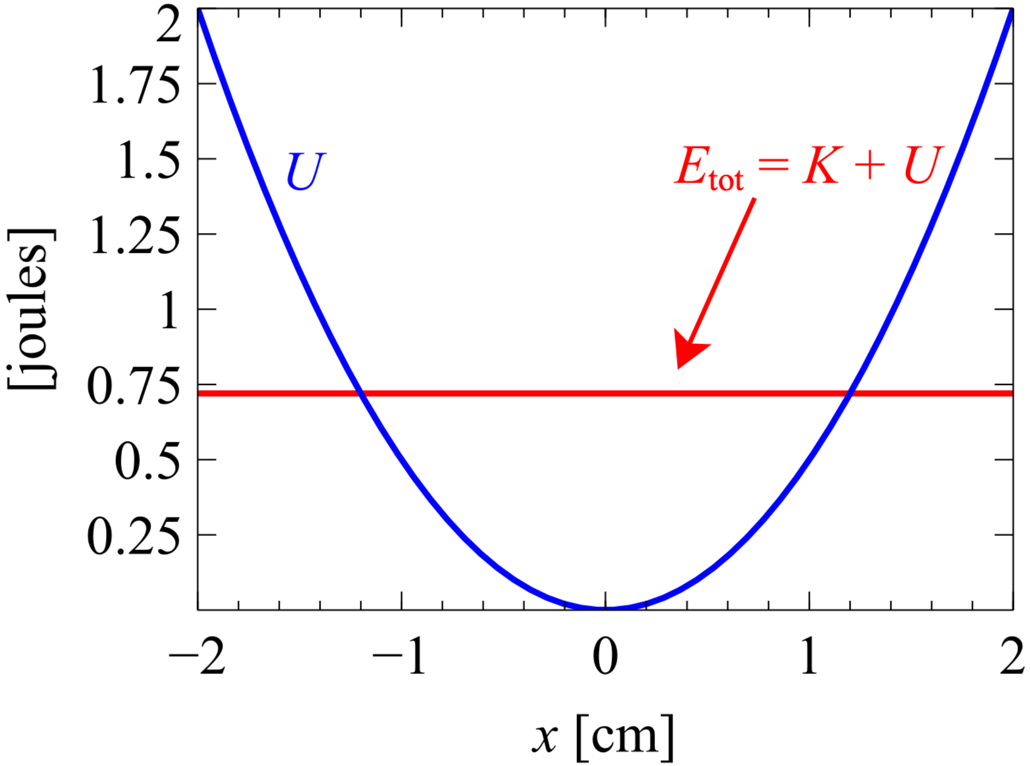
What are potential energy diagrams? (PEDs)

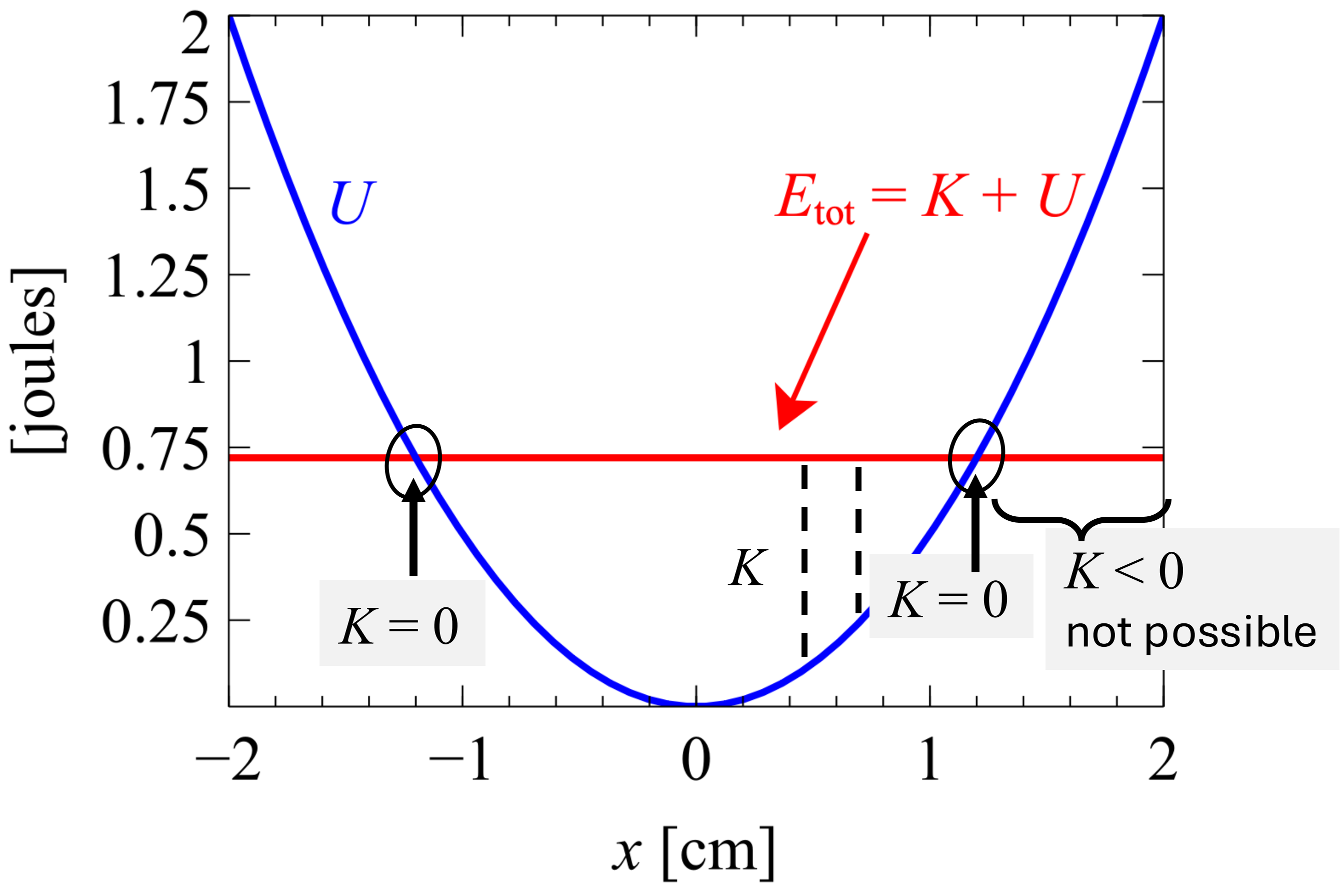


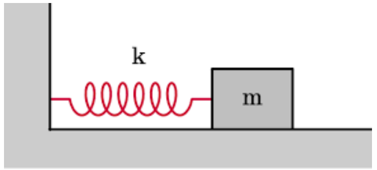
$$U = \frac{1}{2}kx^2$$



Actual total energy (constant) often graphed







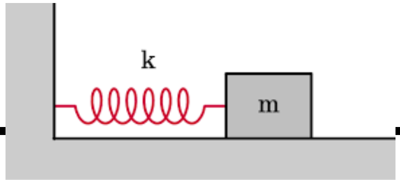
How to think of PEDs

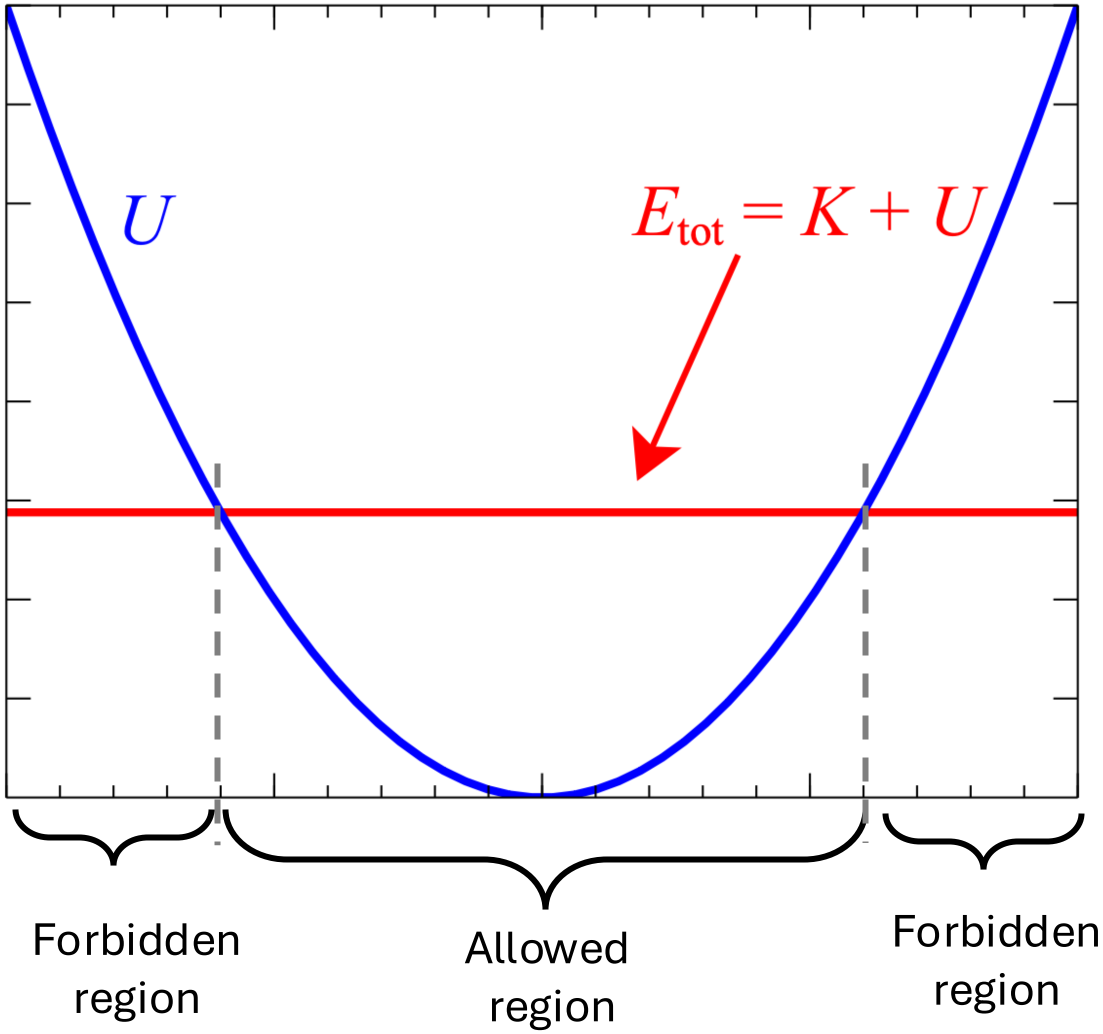
$U = E$ at **turnaround points**

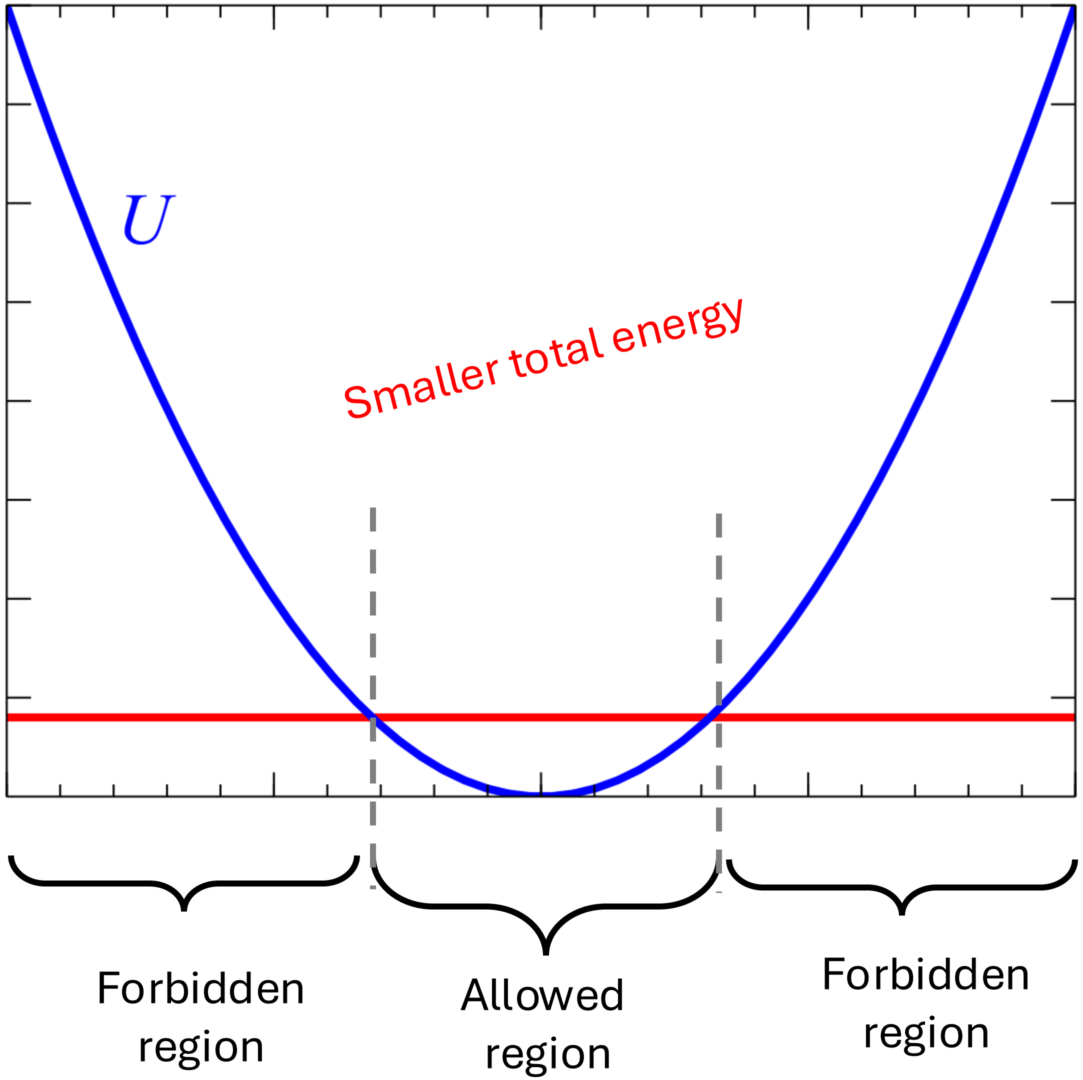
Particle moves back and forth between turnaround points

Classical particle can't be located where $U > E$

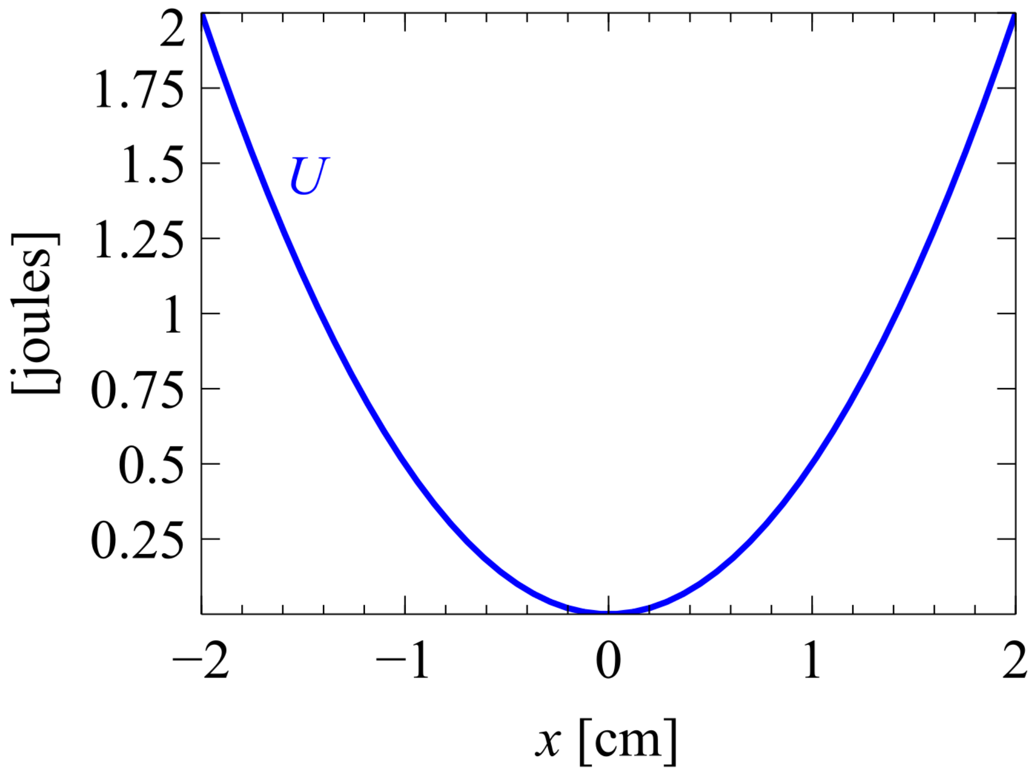
How to think of PEDs classically

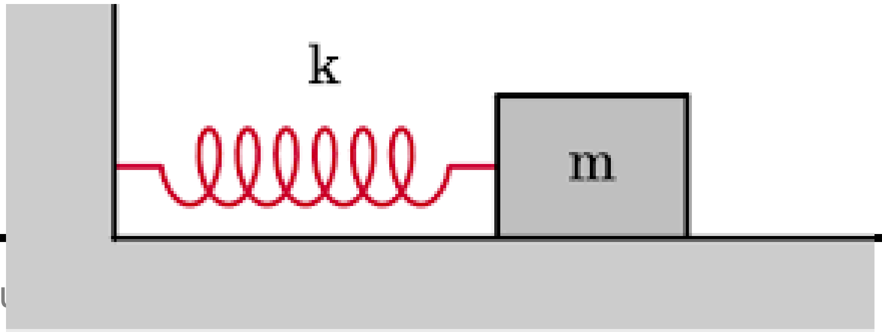






Can also get information about force





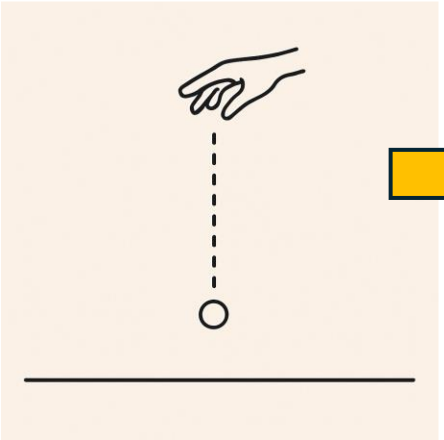
$$U = \frac{1}{2}kx^2$$

$$F_x = -\frac{\partial U}{\partial x}$$
$$= -kx$$

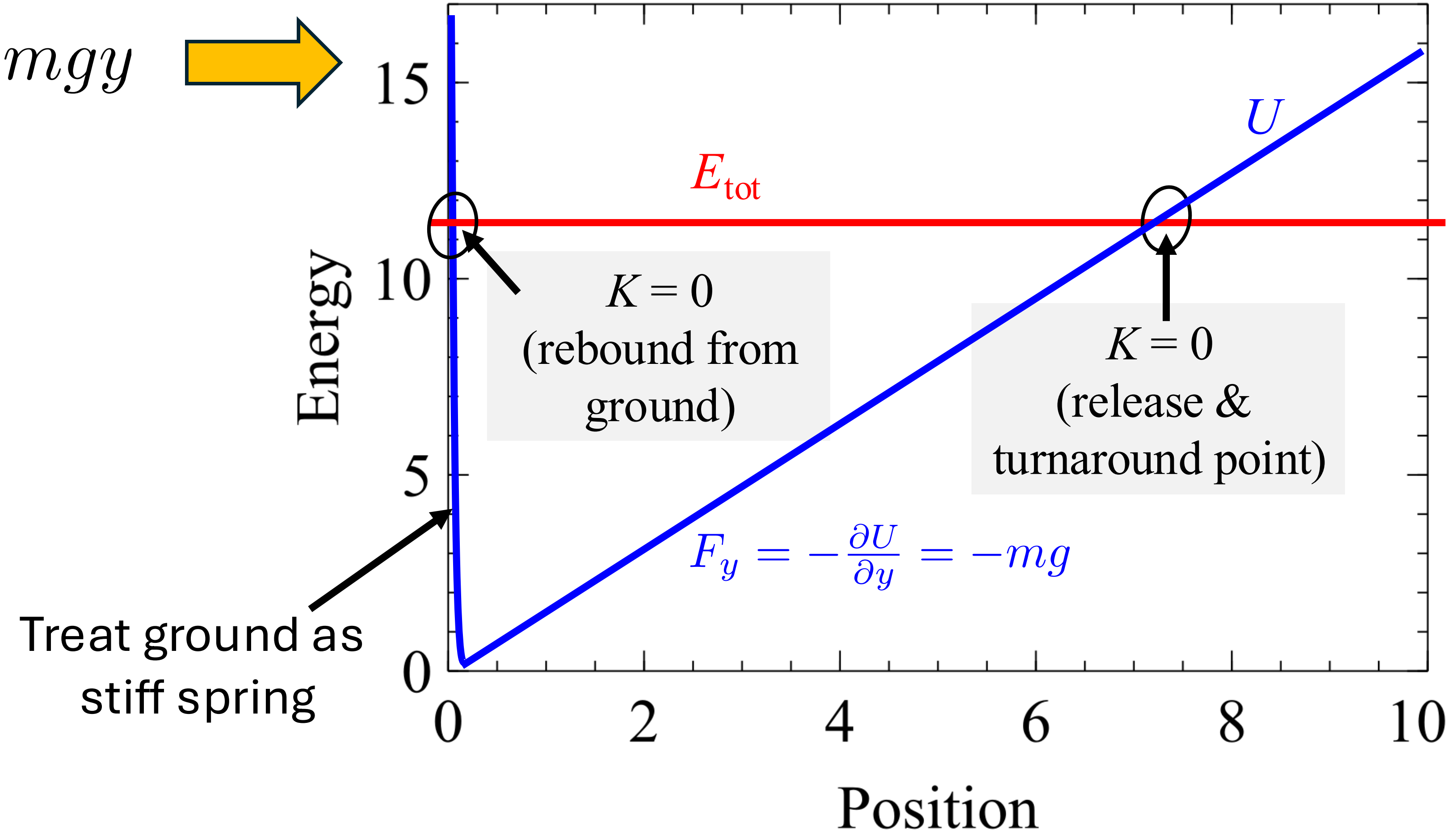
Stretch
$x > 0 : F_x < 0$

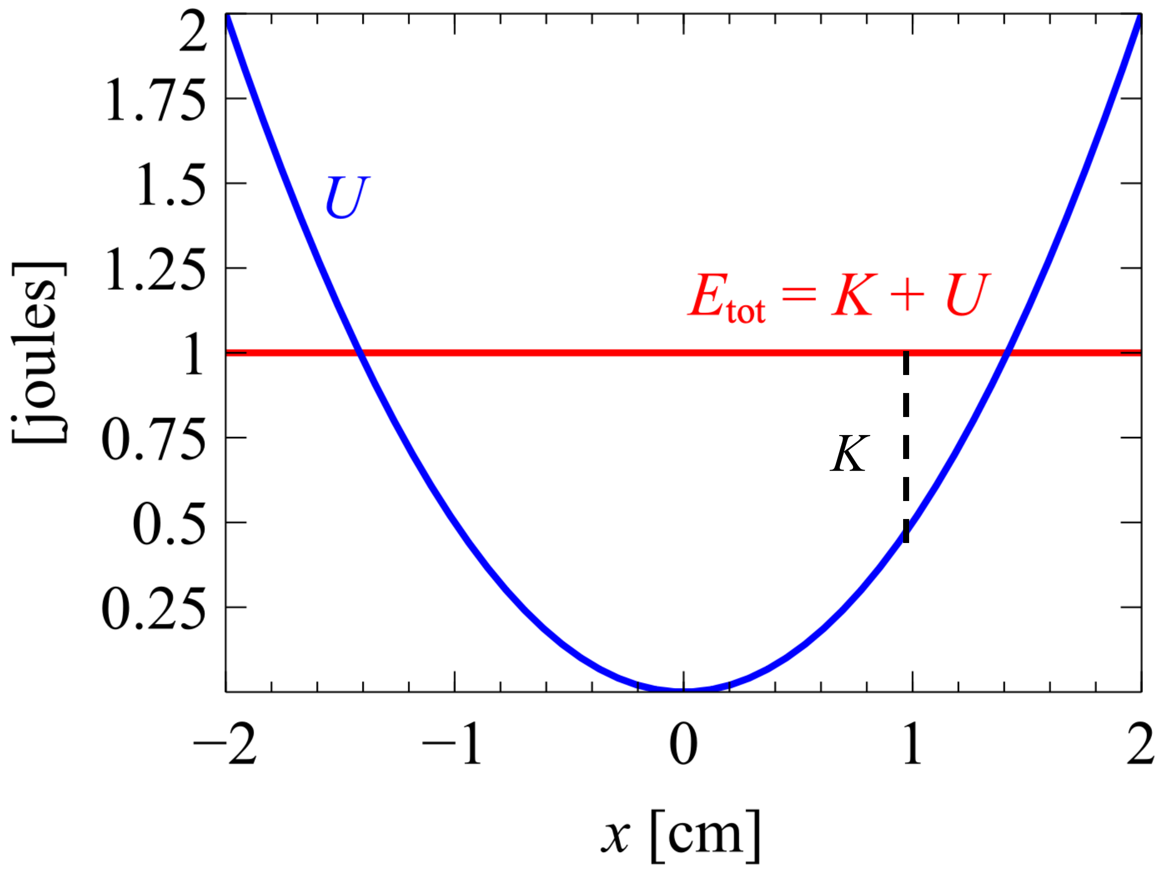
Compress
$x < 0 : F_x > 0$

Conservative force points downhill on PED



$$U = mgy$$


Energy
Position
0
5
10
15
0
2
4
6
8
10
U
E_{tot}
$K = 0$
(rebound from ground)
$K = 0$
(release & turnaround point)
$F_y = -\frac{\partial U}{\partial y} = -mg$
Treat ground as stiff spring



Posed to students

When the particle is at 1.0 cm, which best describes its motion?

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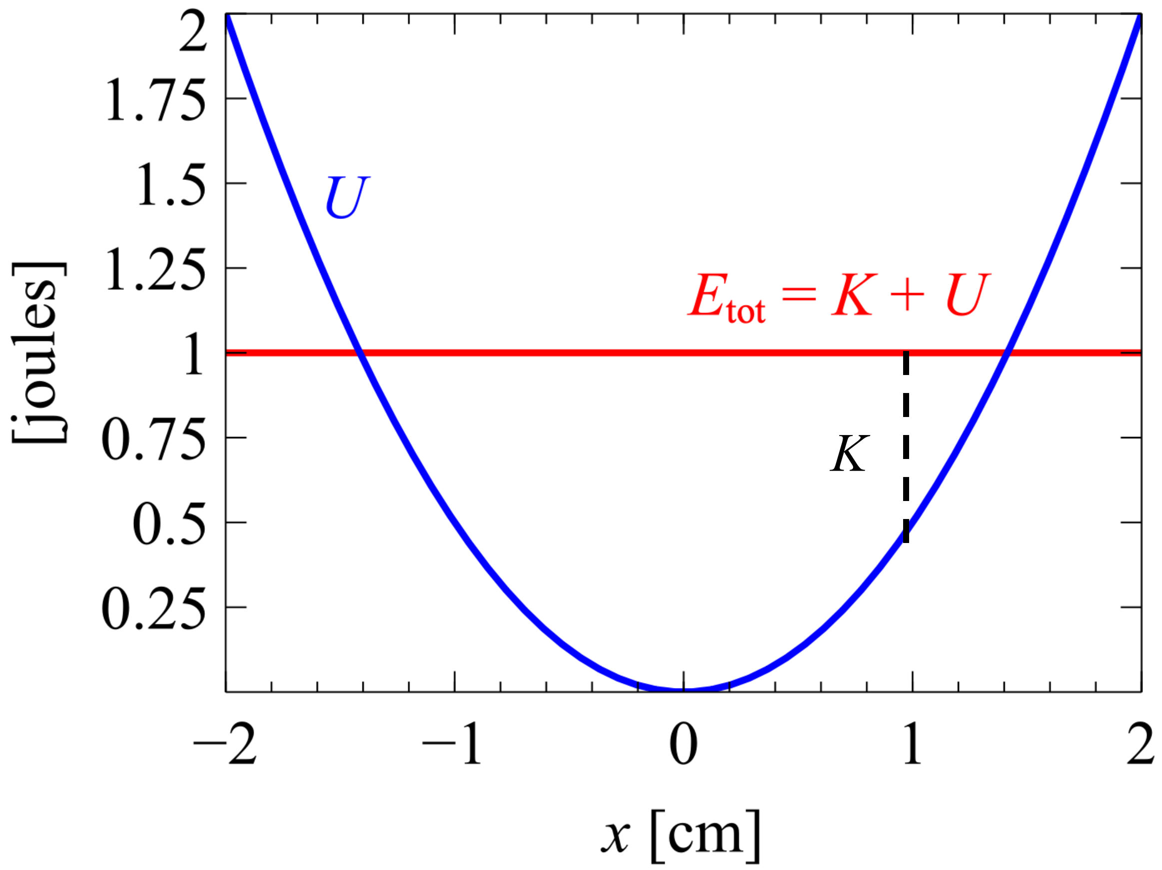

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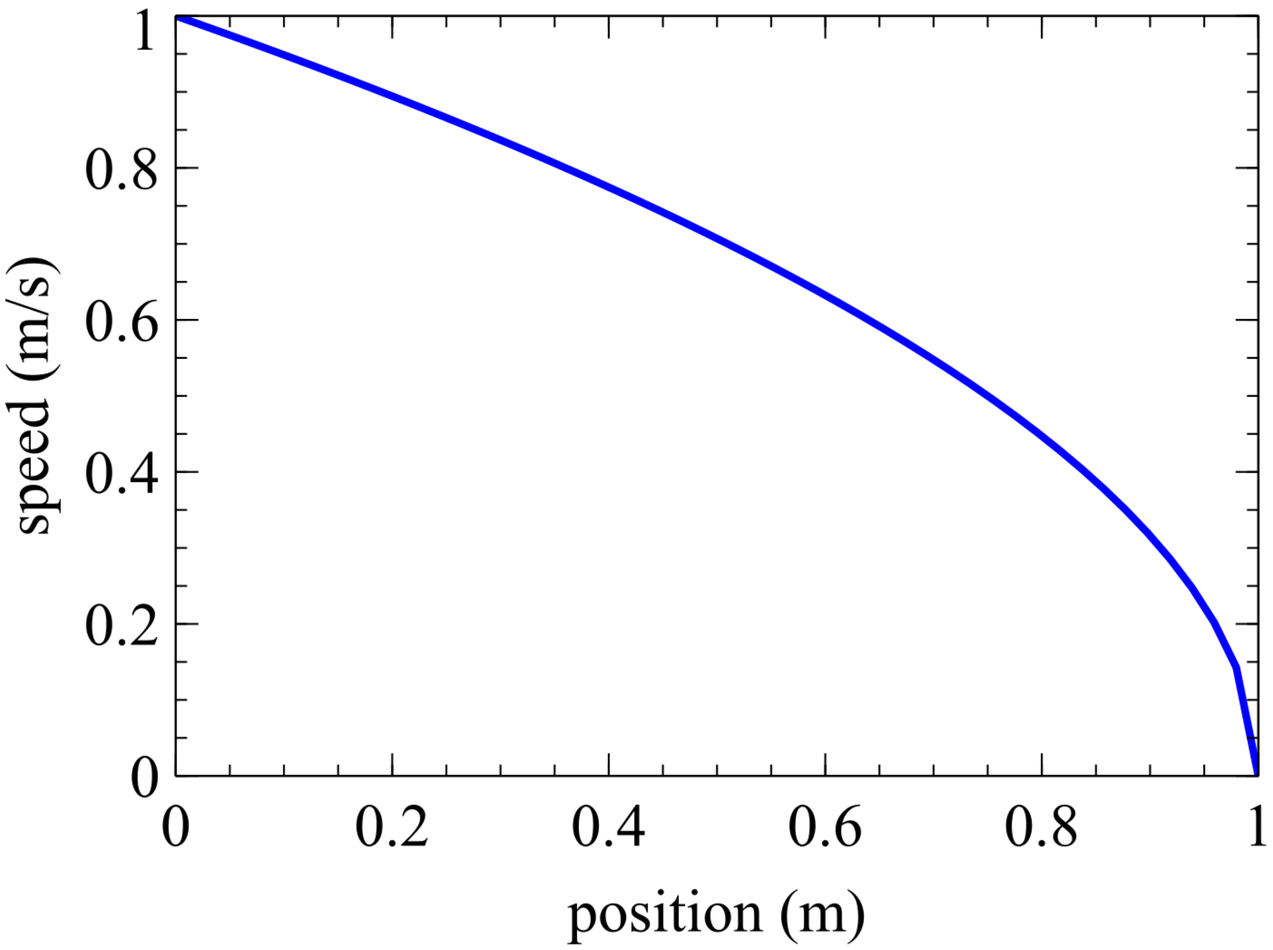
Horizontal axis does not represent time.
Can be moving in +x or –x direction.

A question we'll come back to



Revisiting…

When the particle is at 0.4 m, which best describes its motion?

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- *Slowing down*
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- *Constant speed*

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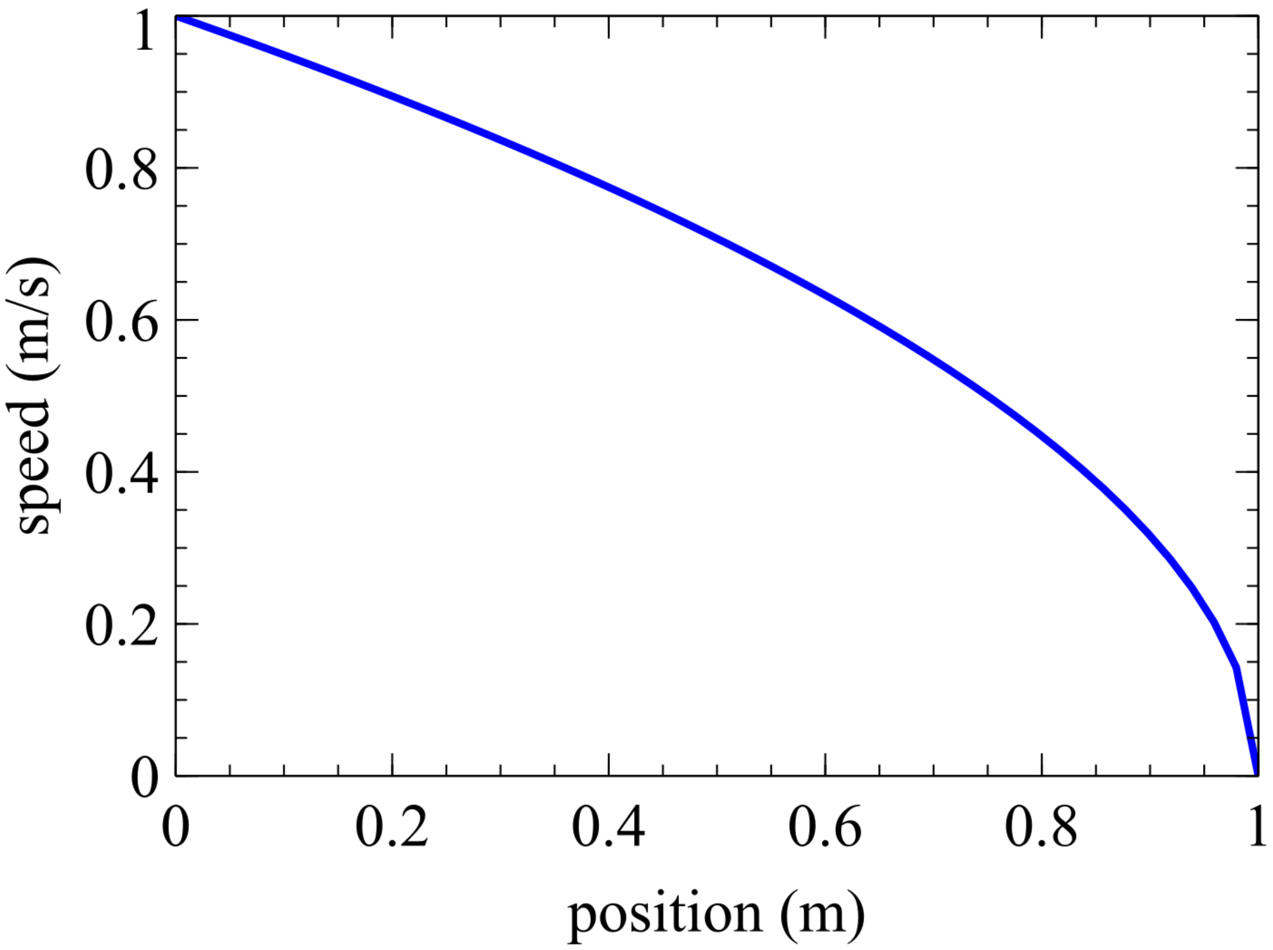

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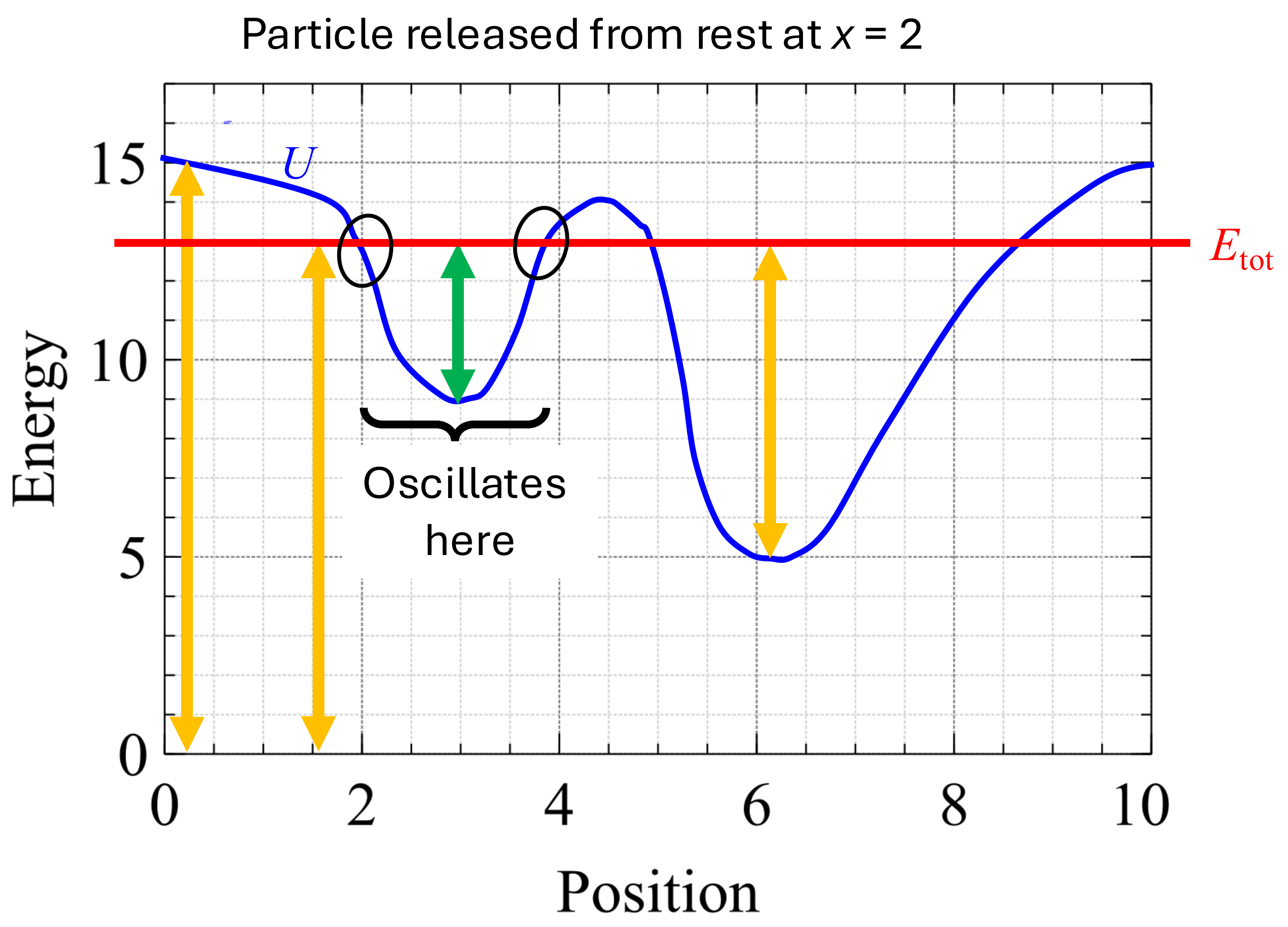
What are common ways that
learners of physics interpret PEDs?



Interpret horizontal axis as time

"At x = 3.75, U increasing so K decreasing."

Overgeneralizing minimization of PE

"At x=3.75, object moving in –x dir, always toward lower PE. So KE increasing."

Ignoring potential barriers (Tunneling?)

"Kmax is 13 J – 5 J = 8 J"

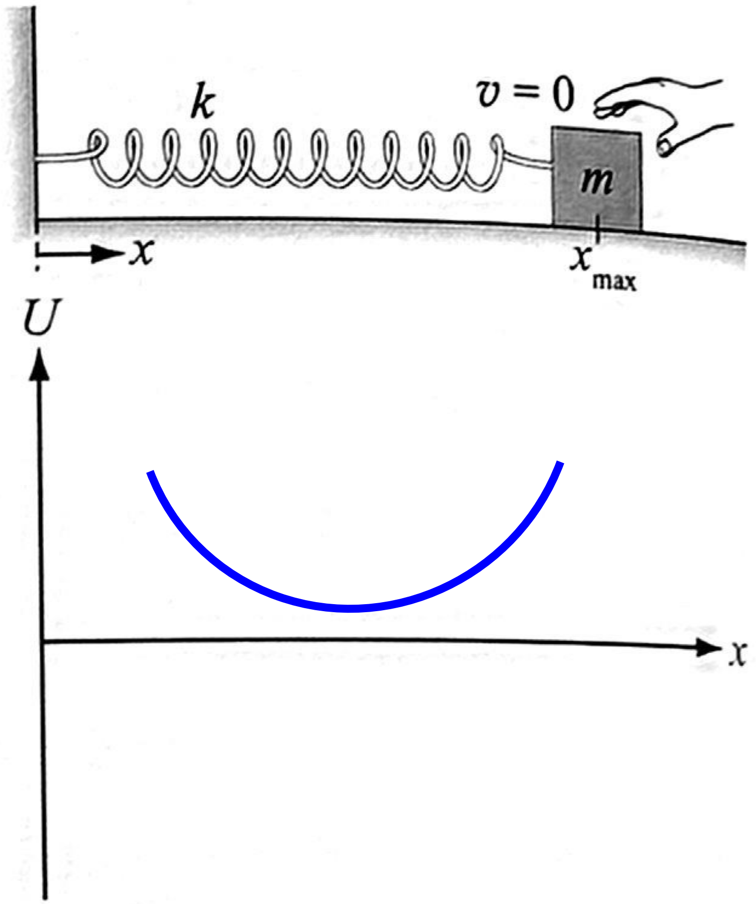
Assuming K_{max} = E_{tot}

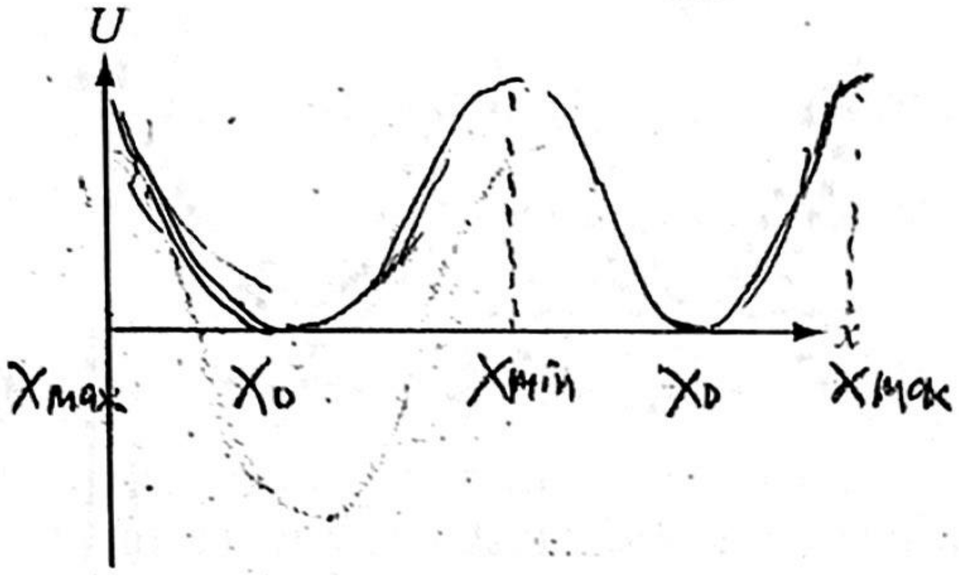
"Kmax is 13 J since that's the total E"

Assuming K_{max} = U_{max}

"Kmax is 15 J since that's the most PE"

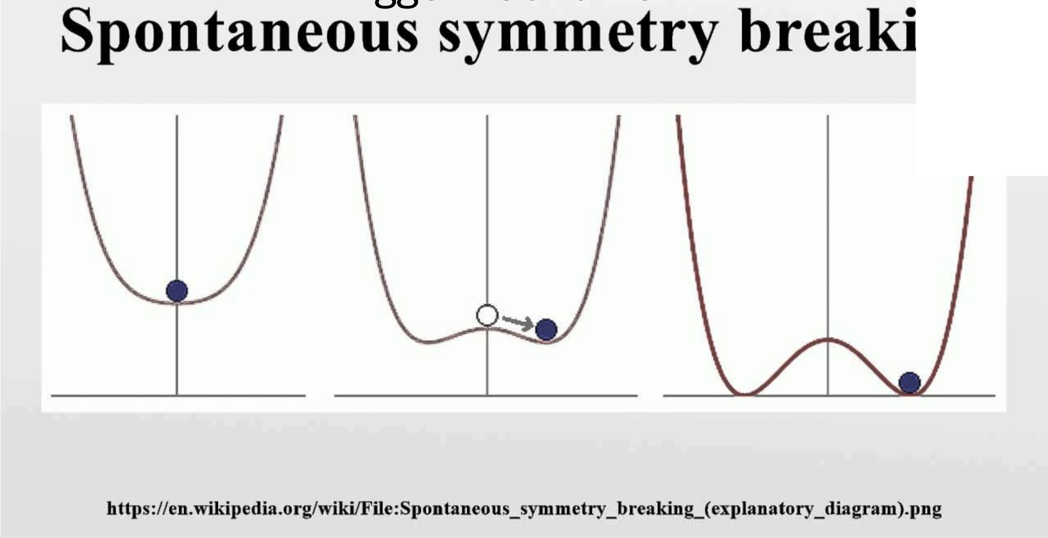
Another manifestation of horizontal axis as time…

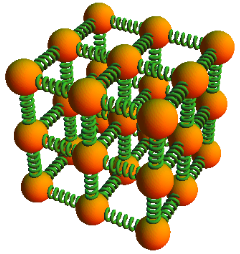




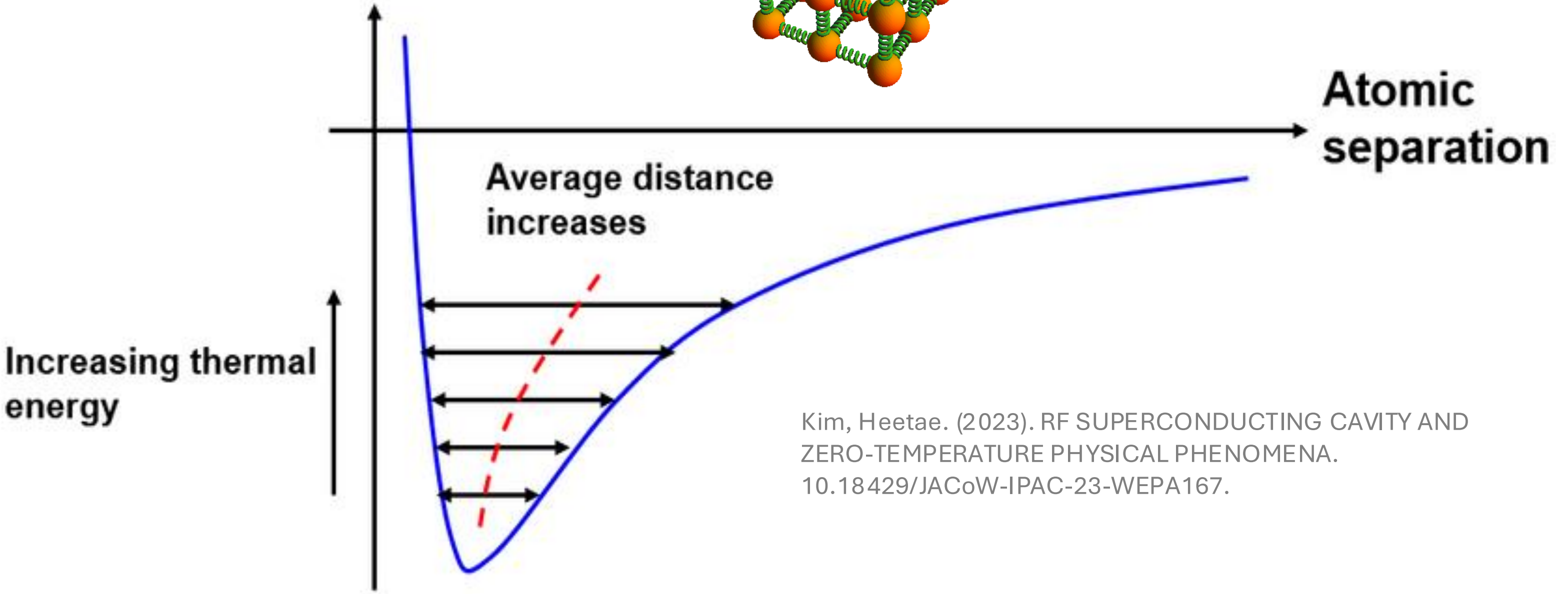
Graph shape is consistent with time,
but horiz axis has position-like labels.

Higgs mechanism





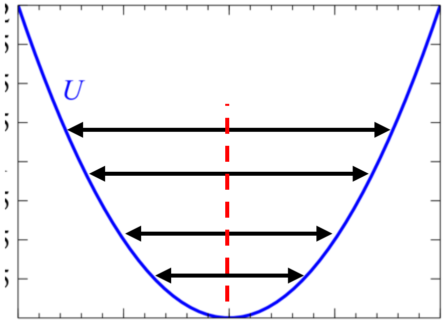
Cartoon model of solid



Kim, Heetae. (2023). RF SUPERCONDUCTING CAVITY AND ZERO-TEMPERATURE PHYSICAL PHENOMENA. 10.18429/JACoW-IPAC-23-WEPA167.

Thermal expansion tells us there is **asymmetry** in the interatomic potential energy.

Symmetric case



Classical potential energy diagrams → Classical probability density → Quantum mechanics

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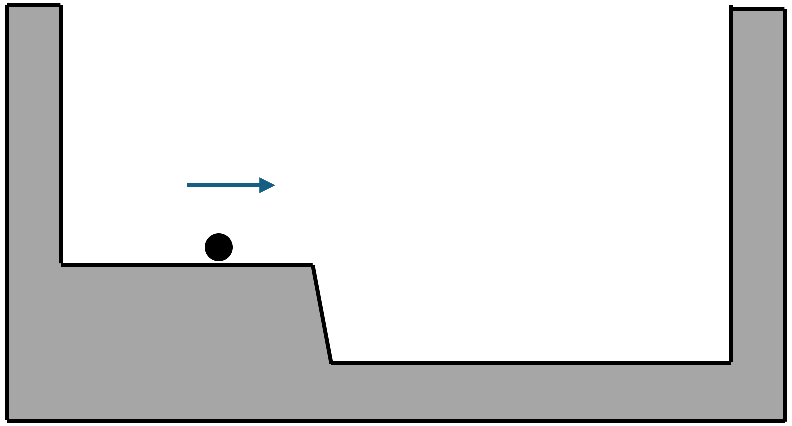
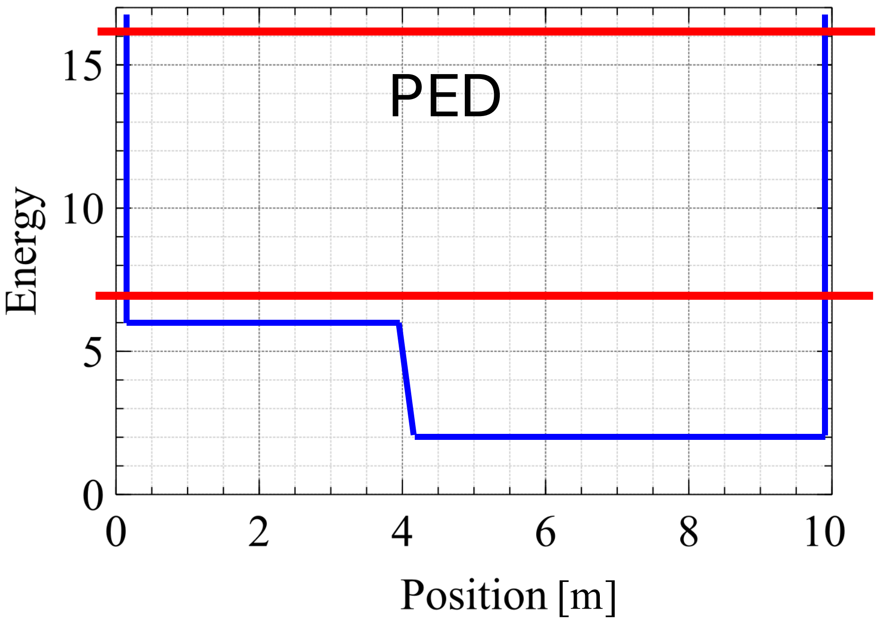


A classical object moves back & forth without dissipative forces. It never leaves the surface of the track and bounces elastically with the walls.

While oscillating, the position of the ball is measured at a **random time.**



Convince yourself:
We *cannot* compare the probabilities of finding the ball in the upper & lower levels.
(Can't compare times spent in regions.)

For the sake of argument:
Suppose the times spent in the two regions are 6 s (upper) and 4 s (lower).

$$P_{\text{upper}} = \frac{6\,\mathrm{s}}{6\,\mathrm{s} + 4\,\mathrm{s}} = 60\%$$

If length of upper region is 4 m…

…probability of finding the ball in each 1 m region is…

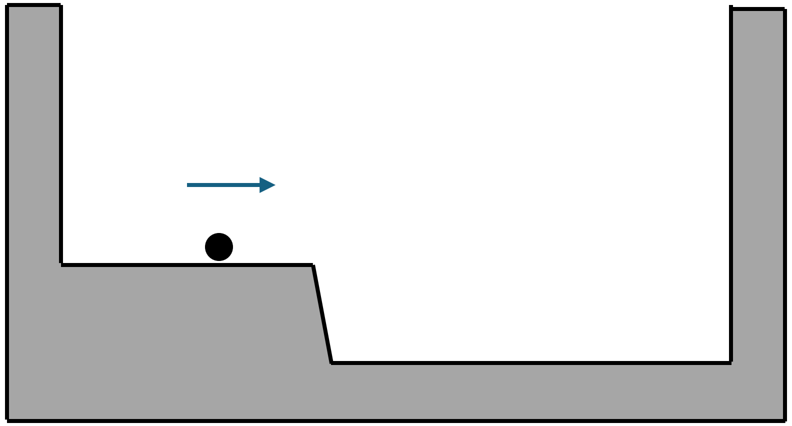
$$15\% \text{ per } 1 \text{ m}$$

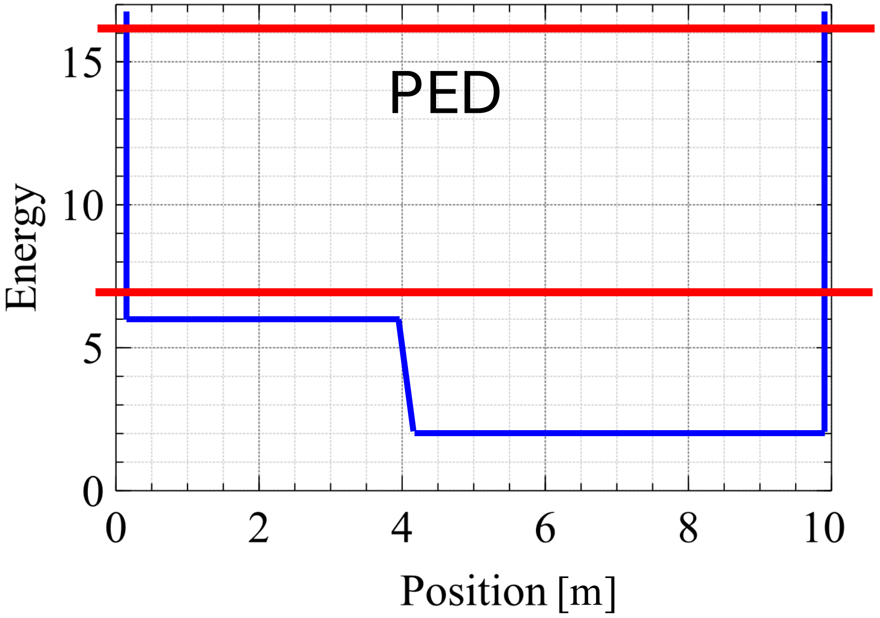
$15\,\%/\mathrm{m}$ is called the **probability density ρ**

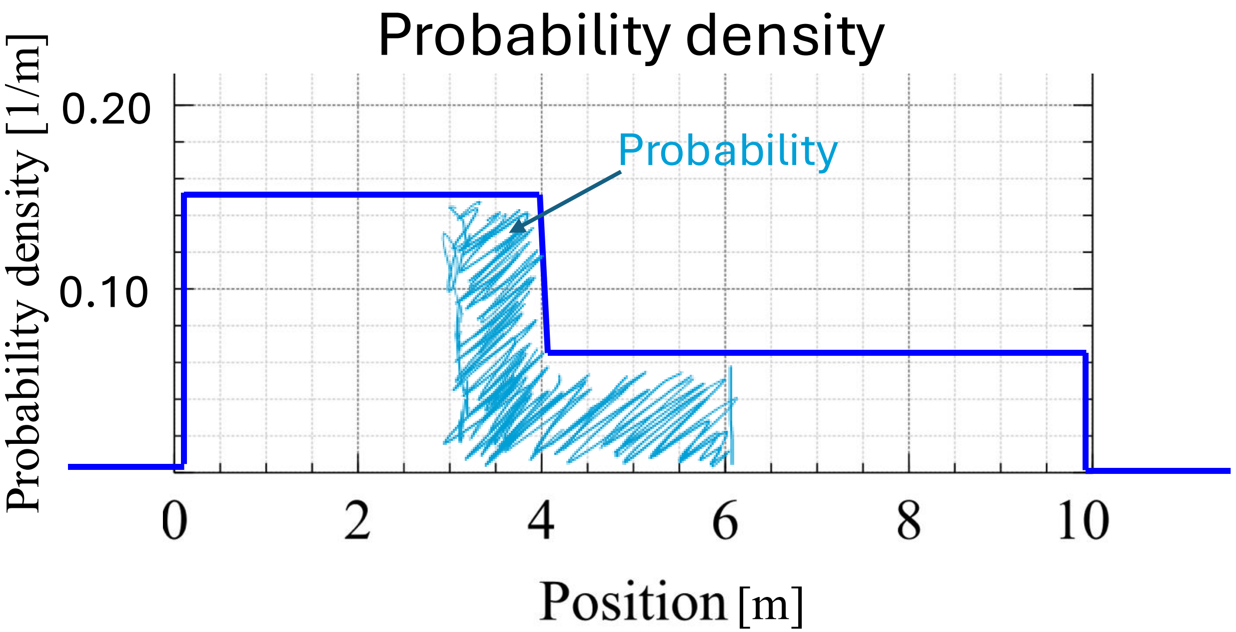
$$\rho(x) \equiv \frac{\mathrm{d}P}{\mathrm{d}x}$$

Probability per unit length

How to think of probability density







$$\rho(x) \equiv \frac{\mathrm{d}P}{\mathrm{d}x}$$

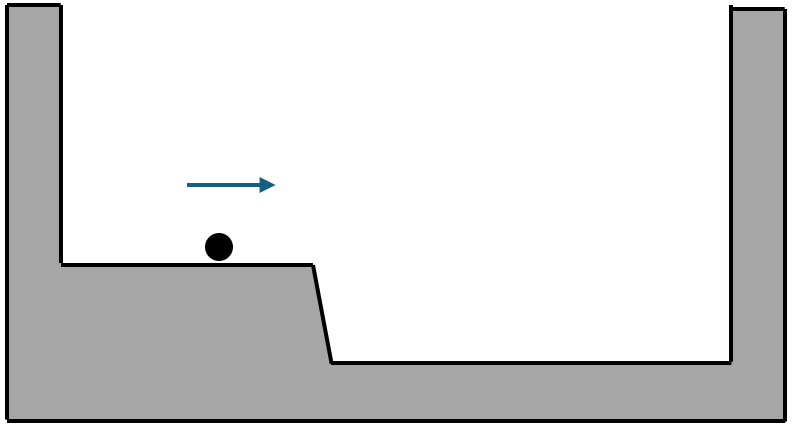
Probability per unit length

$$P(a < x < b) = \int_a^b \rho(x)\,\mathrm{d}x$$

$$\int_{-\infty}^{\infty} \rho(x)\,\mathrm{d}x = 1$$

Normalization condition

How to think of probability density

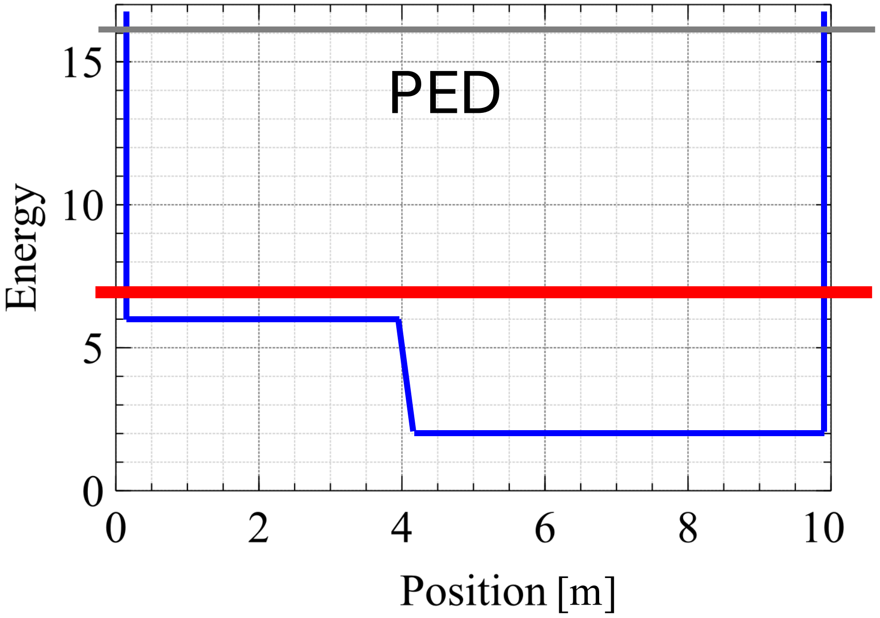


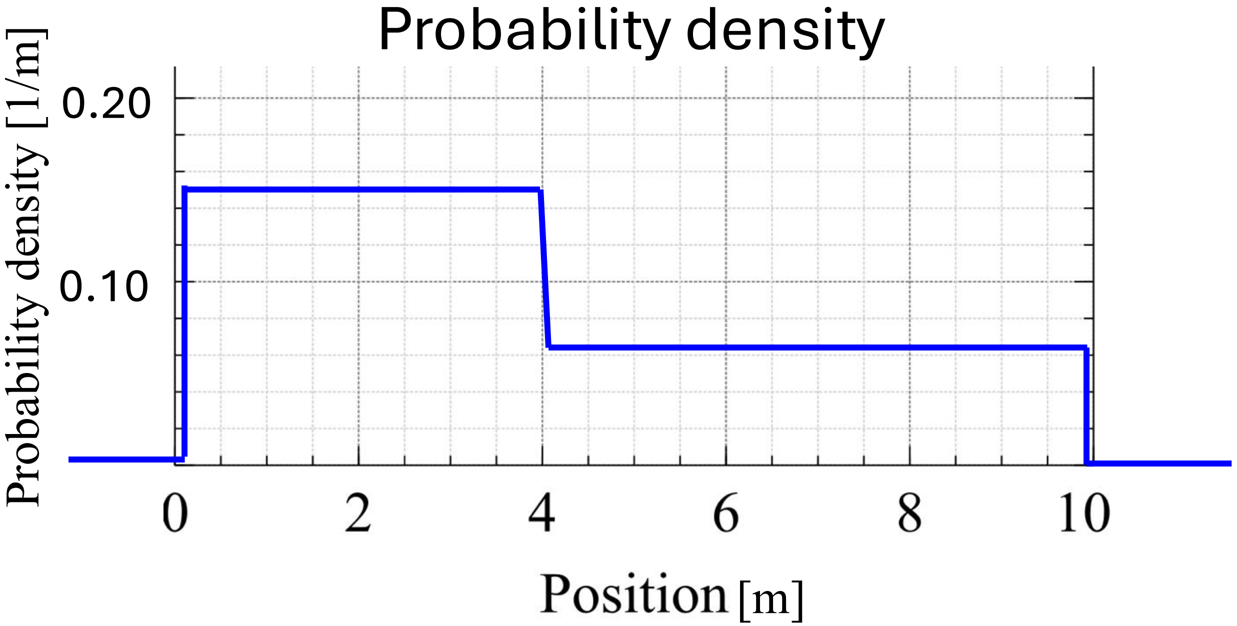
Can't generally compare probabilities P in each region
(without explicit speed or time info)

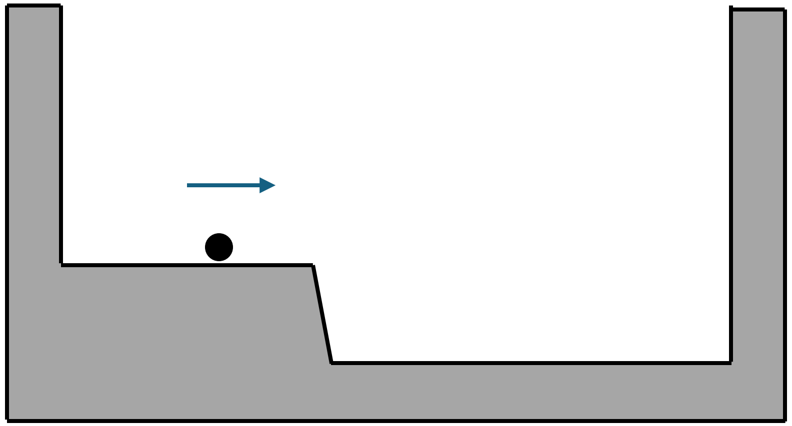
But we **can** compare probability densities ρ

$$\rho(x) \equiv \frac{\mathrm{d}P}{\mathrm{d}x}$$

Upper region:
Slower in each unit length
→ More time in each unit length
→ Greater prob in each unit length







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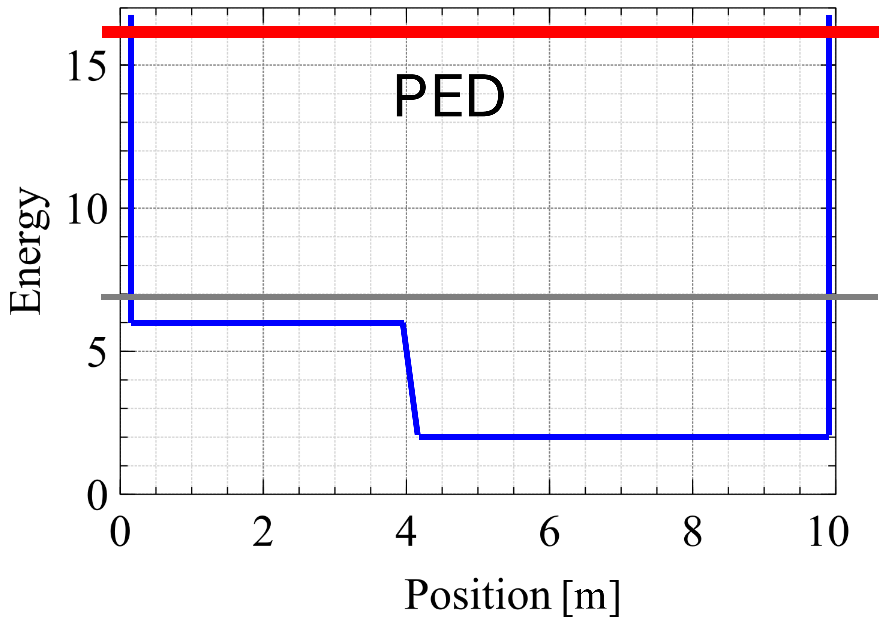

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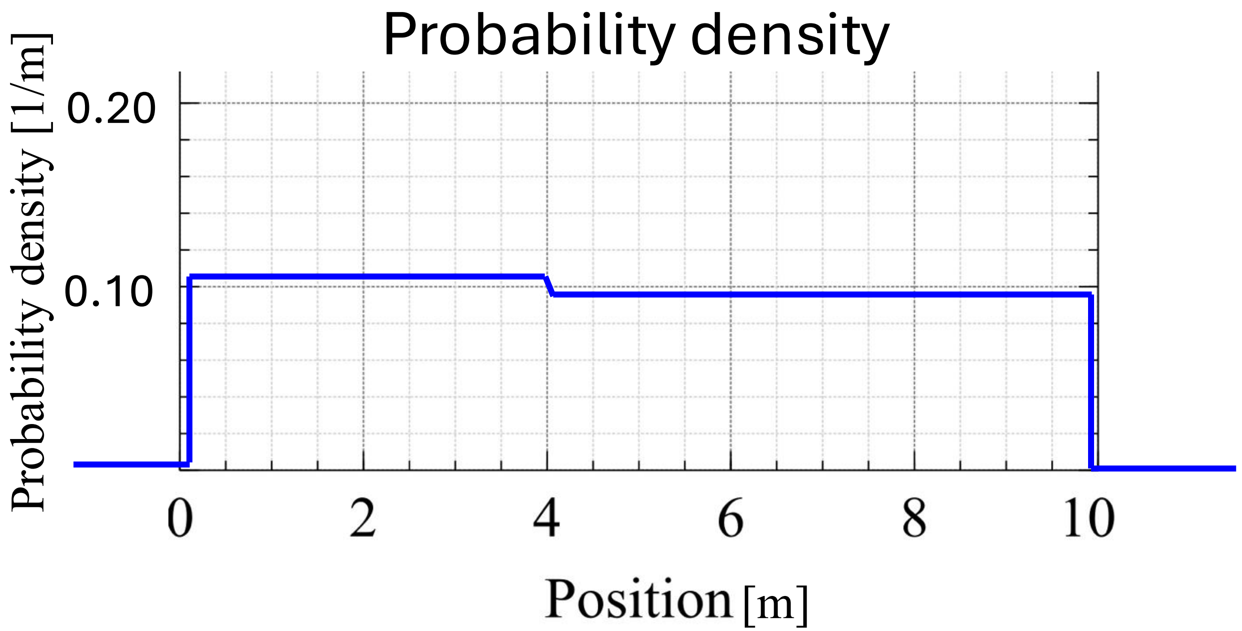
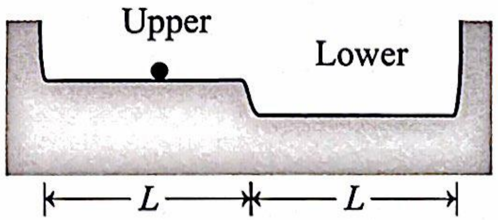


What are common ways that learners of physics think about probability & probability density?

Ball moves back and forth in frictionless track.

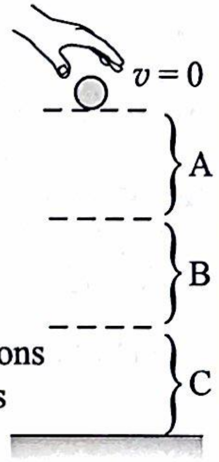
Position is measured at a random time.



- Rank probabilities in regions
- Rank probability densities

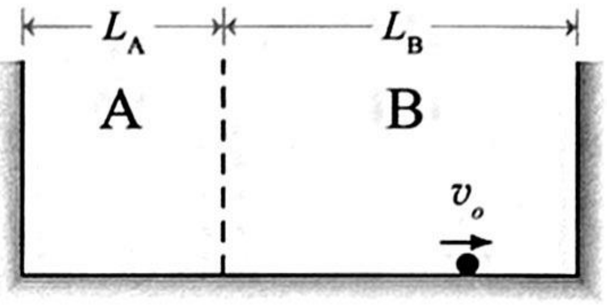
Ball returns to same height after each bounce.

Position is measured at a random time.



- Rank probabilities in regions
- Rank probability densities at centers of regions

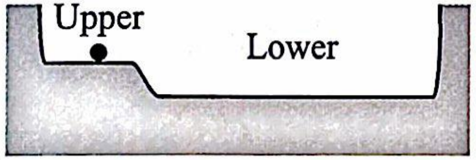
Picture taken at random time



- Rank probabilities in A and B.
- Write expression for probability in region A.

Ball moves back and forth in frictionless track.

Position is measured at a random time.



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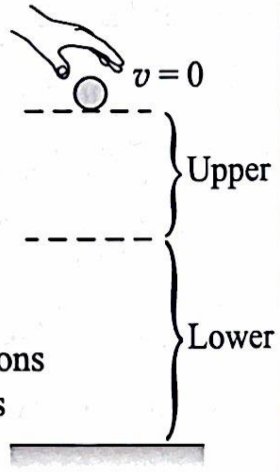
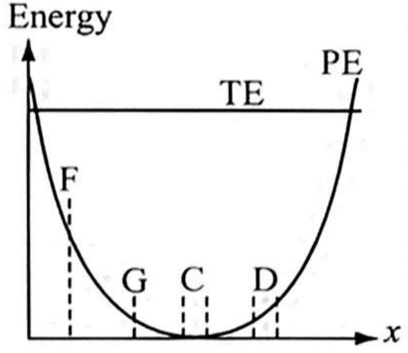
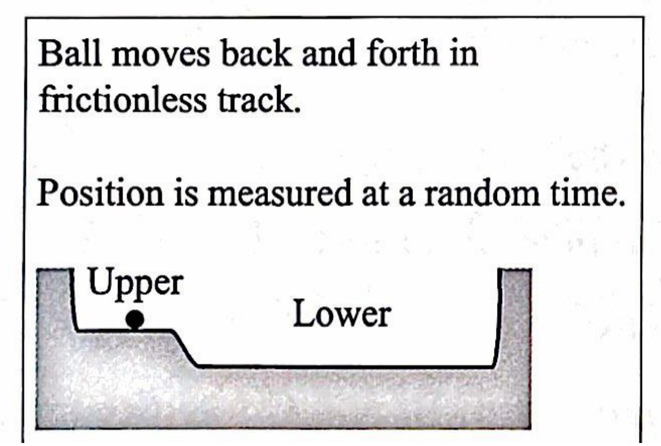


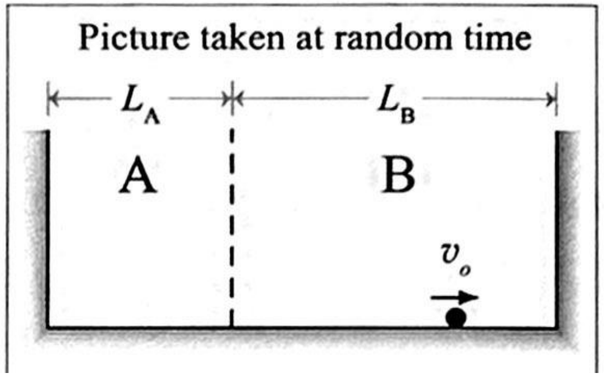
- Rank probabilities in regions
- Rank probability densities at centers of regions

Rank the probabilities of finding the center of the object exactly at points F and G. If either probability is zero, state so explicitly.



How learners of physics think about probability / probability density.





Probability

Rank only by size

"Upper region is smaller, so less chance to catch object there."

Rank only by speed

"Moves slower in upper region, so more time, so greater chances of seeing object"

Competing effects cancel out

"Upper region is smaller but object moves slower. Probabilities must be the same."

Probability density

Not distinguishing between P and ρ

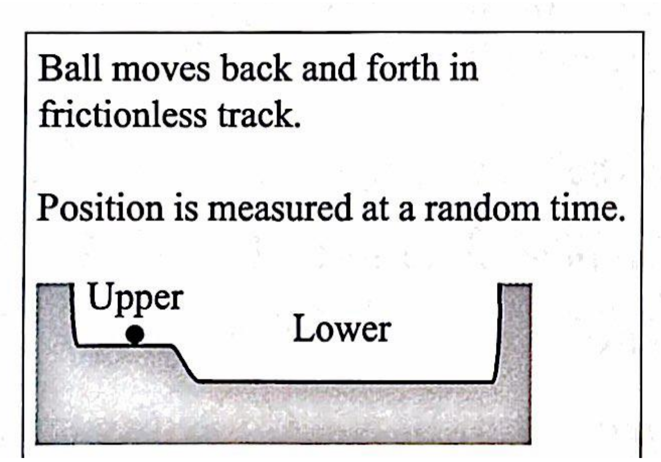
"Greater size means greater chance."

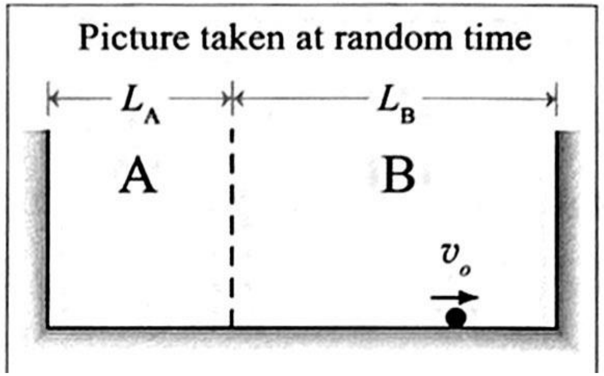
Rank by inverse size

"We divide by length to get density, so greater density in smaller regions"

Ability in mass or charge context does not imply ability in probability context

How learners of physics think about probability / probability density.





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↓

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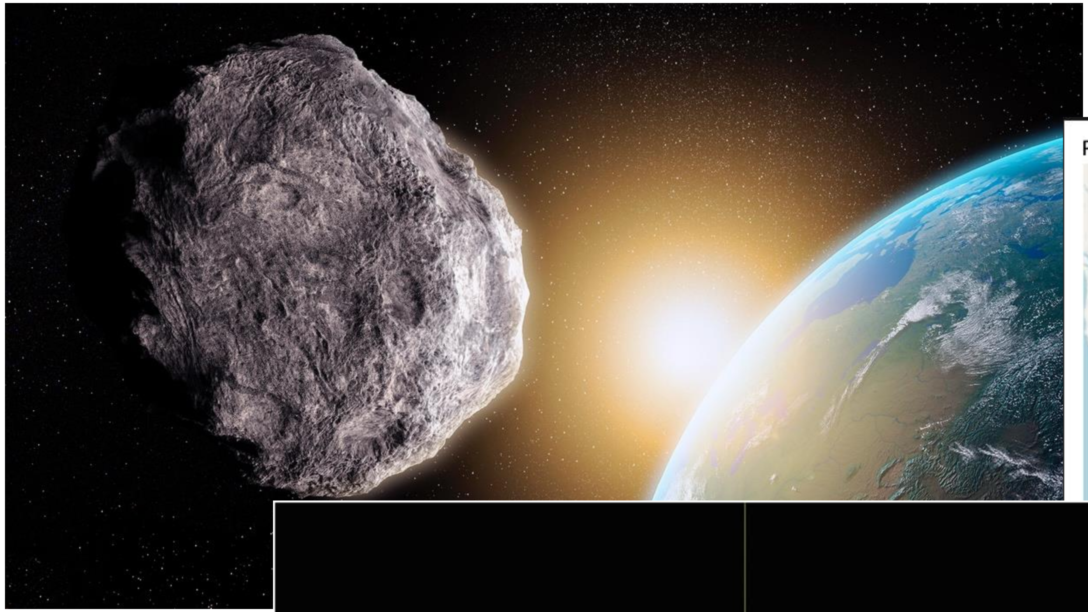
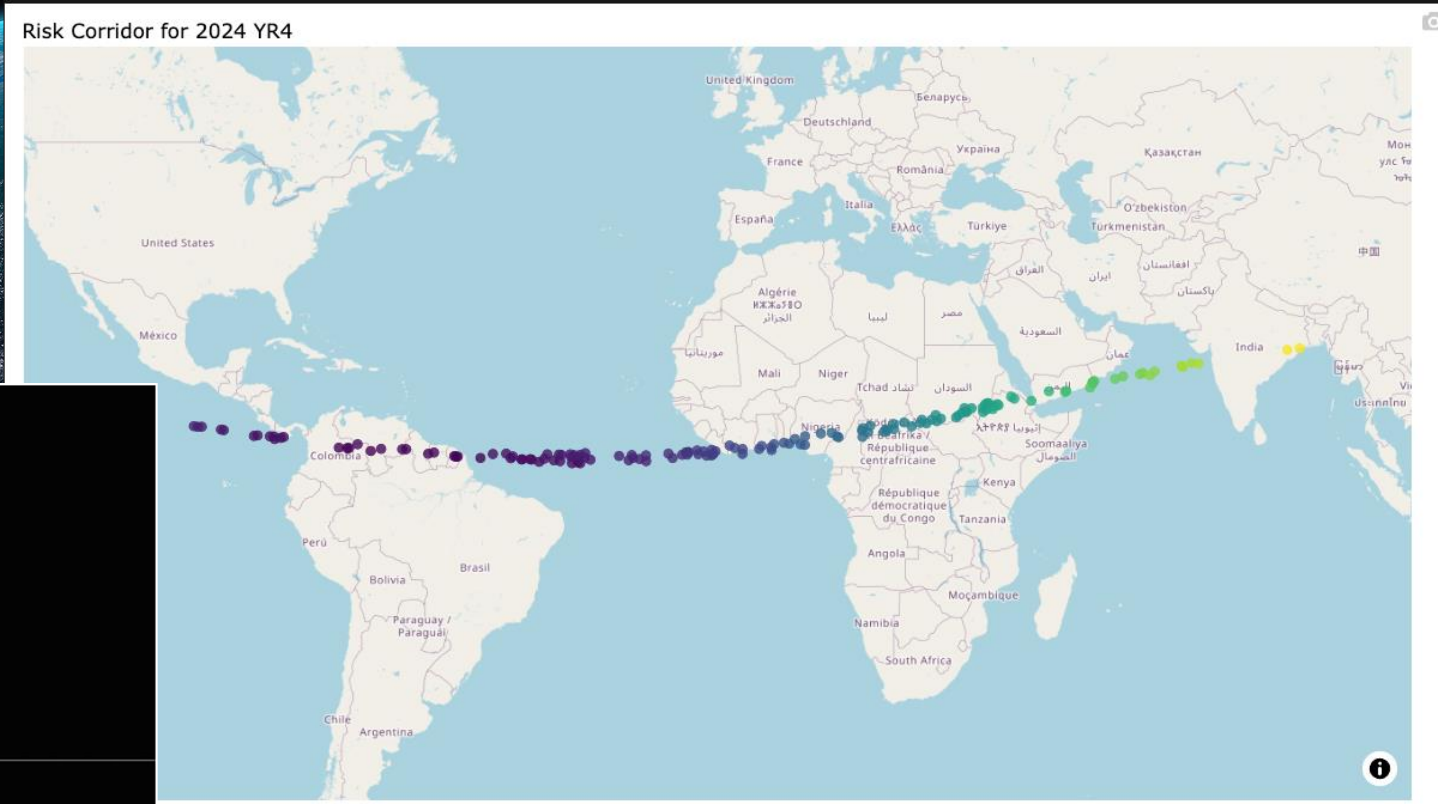
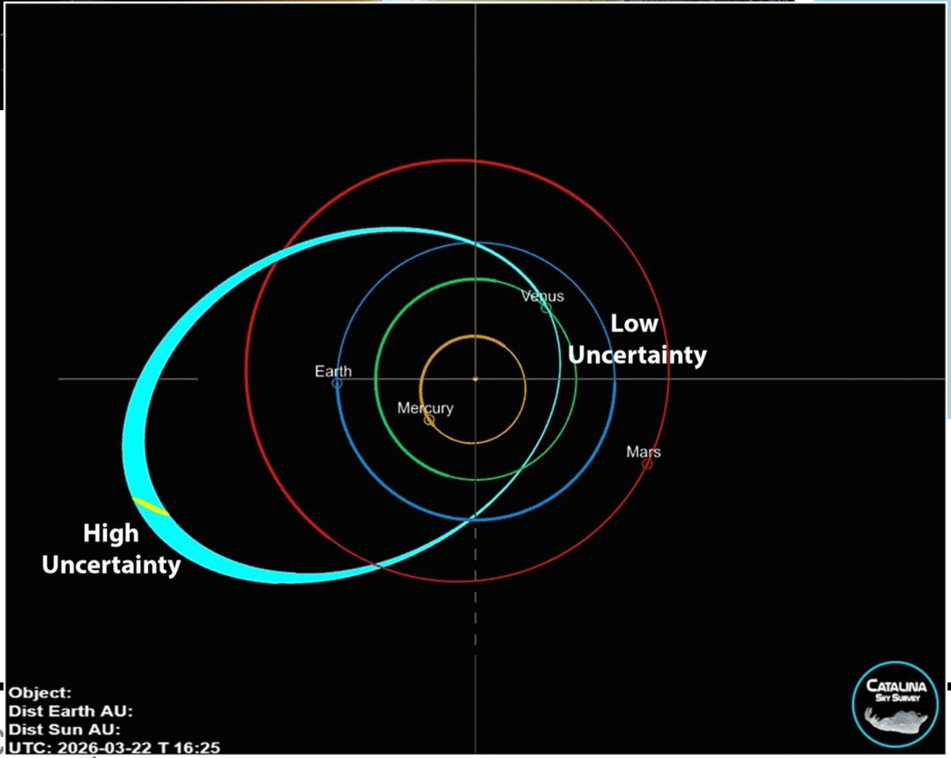
Aside: Classical probability densities is real life







Classical potential energy diagrams

Classical probability density

Quantum mechanics

What are they?
How should we think of them?
What is difficult about learning them?

How can our classical knowledge help us?

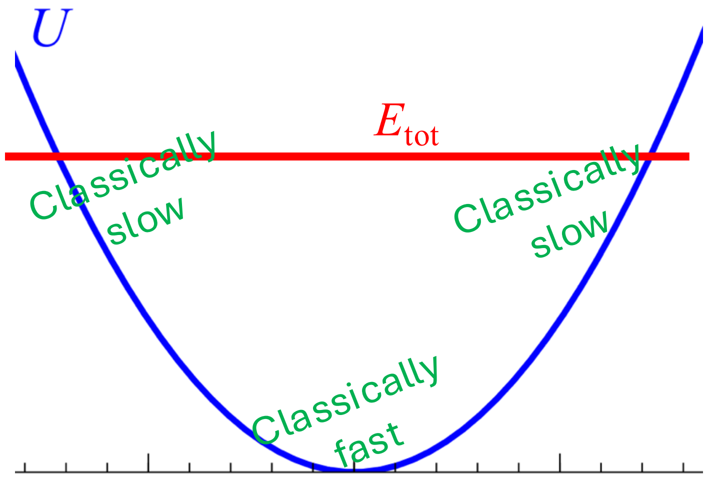
QM energy eigenvalue equation

(Time-independent Schrodinger eqn)

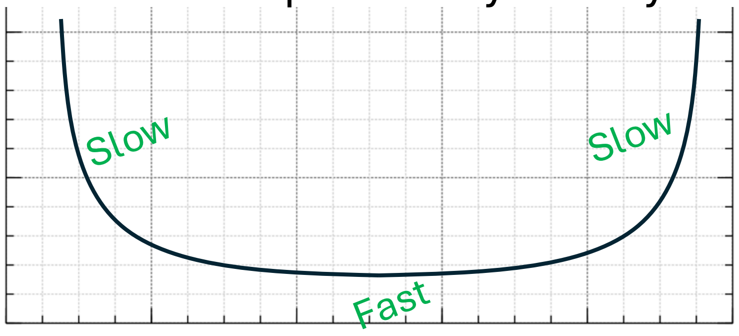
$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi(x) = \underbrace{\left(E_{\text{tot}} - U(x)\right)}_{\text{Classical KE}}\psi(x)$$

Given E_{tot} *and* $U(x)$, eqn can be solved to determine wave function $\psi(x)$

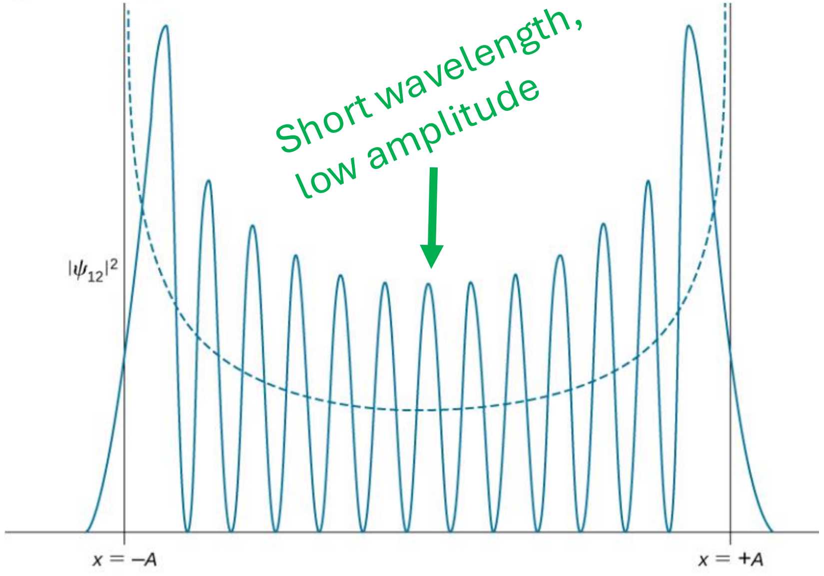
Draw same PED for classical & QM



Classical probability density



QM probability density (solid)



Mathematical consequences (won't prove):

- Classically allowed → ψ is 'wavy' (sinusoidal or sinusoidal-like)
  Classically Fast → ψ *has short wavelength and low amplitude*
- Classically forbidden → ψ is exponential or exponential-like

Connection of ψ to reality:

Probability density $\rho(x) = |\psi(x)|^2$

So ρ has similar behavior to ψ

Correspondence principle:
At high enough energies, the [spatially averaged] quantum & classical ρ look similar

Diagram I want my PHYS 225 students to understand

Horiz axis for ψ

Horiz axis for PED

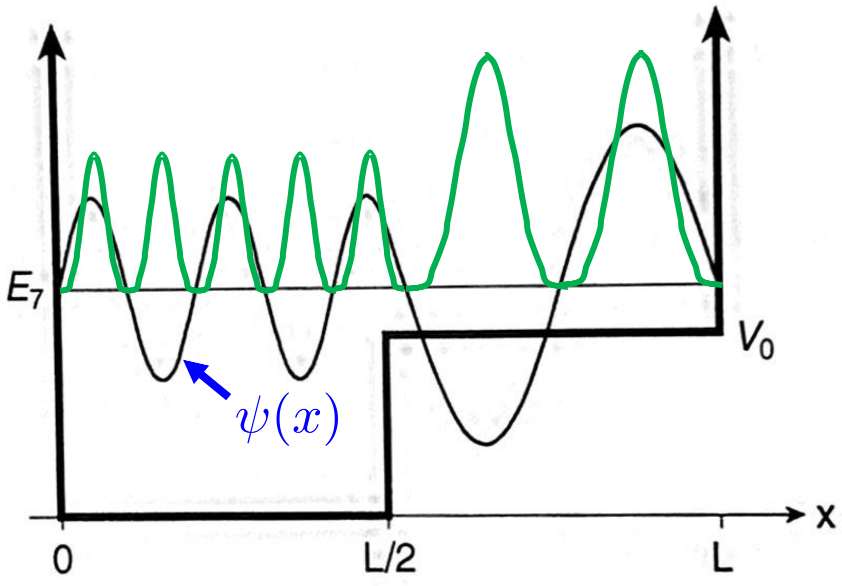
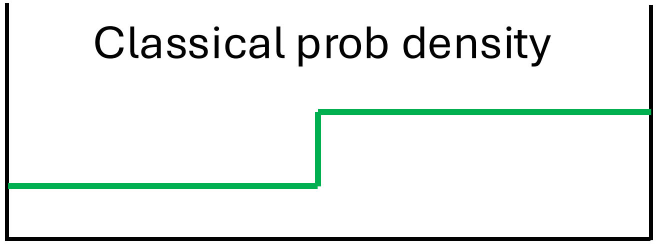


FIGURE 5.27 An energy eigenstate of the asymmetric square well.

Classically, in the right region:

Prob density is higher (because moves more slowly)



Thus, quantum mechanically, in the right region:

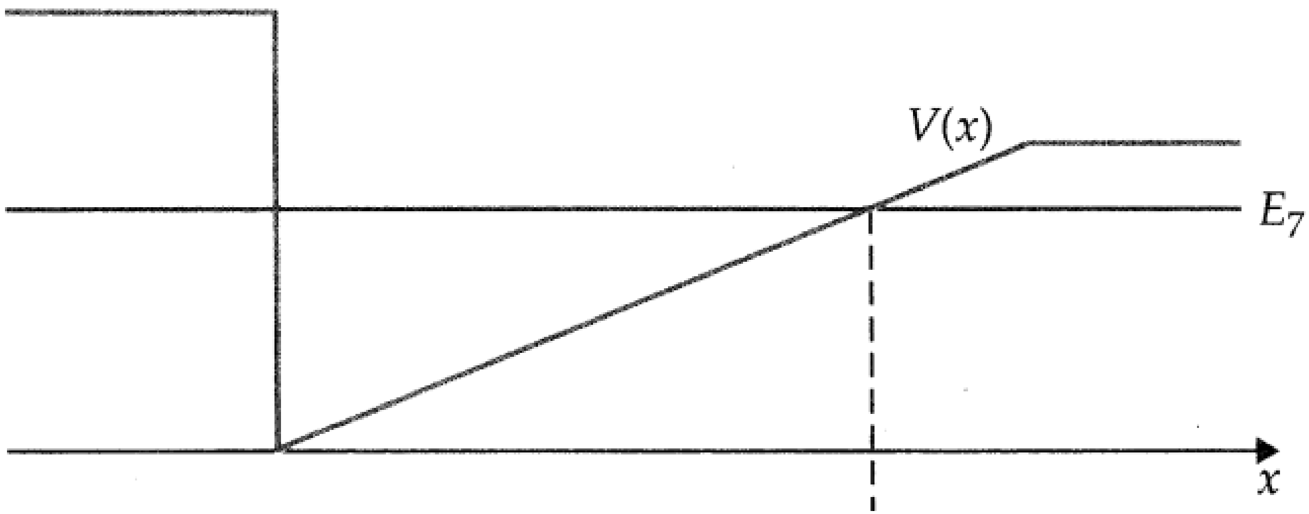
Wave function has larger amplitude & longer wavelength

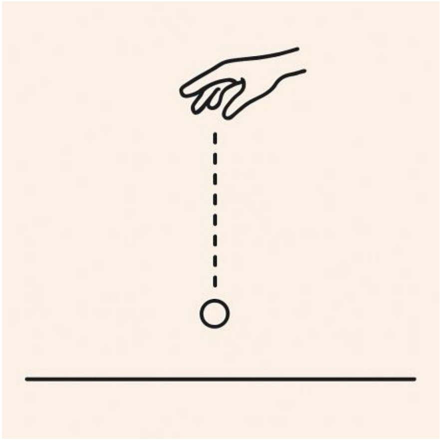
$\rho(x) = |\psi(x)|^2$ has similar behavior

Probability density has larger amplitude & longer wavelength

Fun at home tonight!

PED





Bored at home tonight?

What would classical probability density look like?

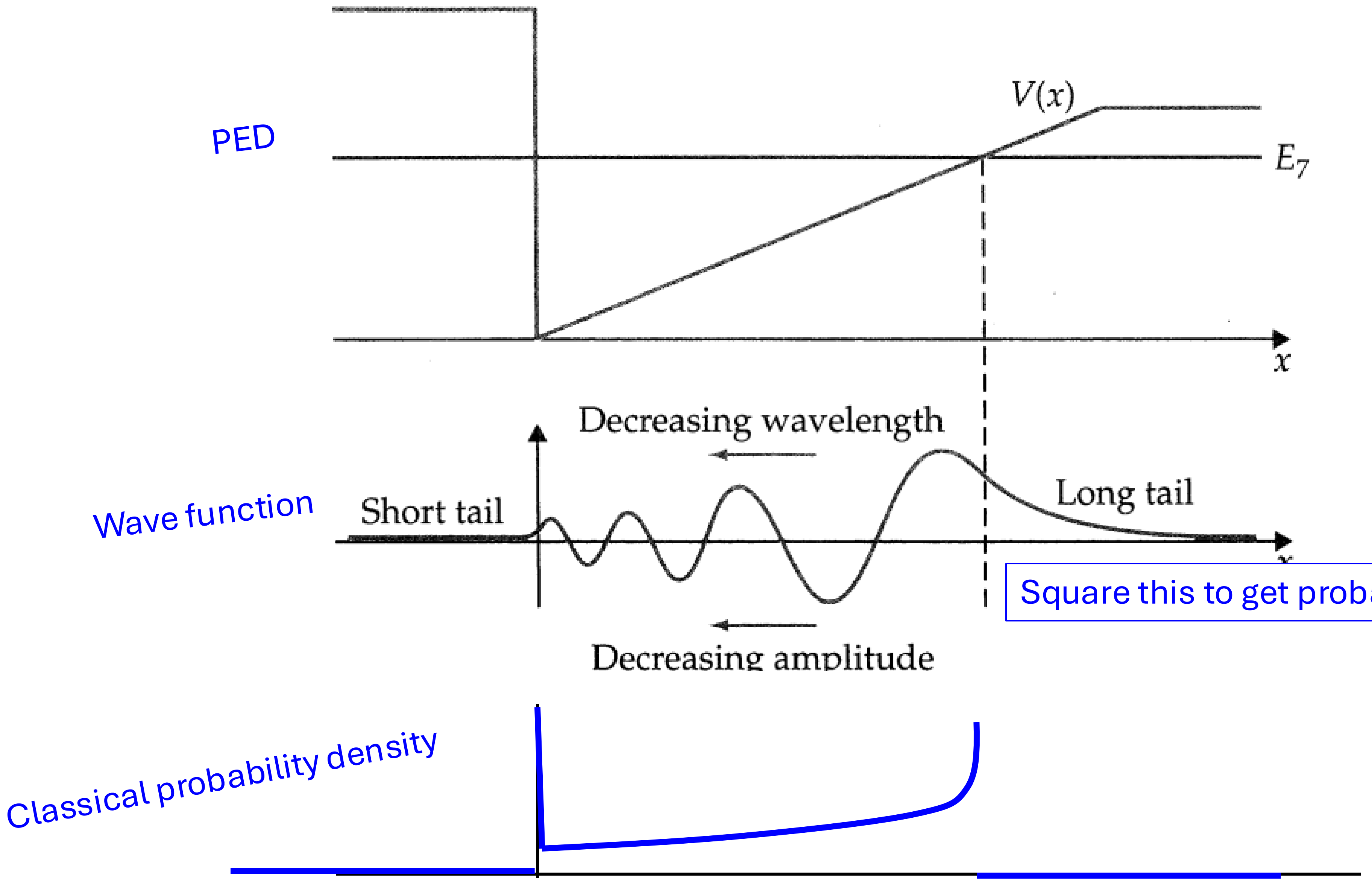
What would quantum probability density look like?

How are they similar? Different?

That’s it!

“Solutions”

Or close enough to them…



Square this to get probability density for QM