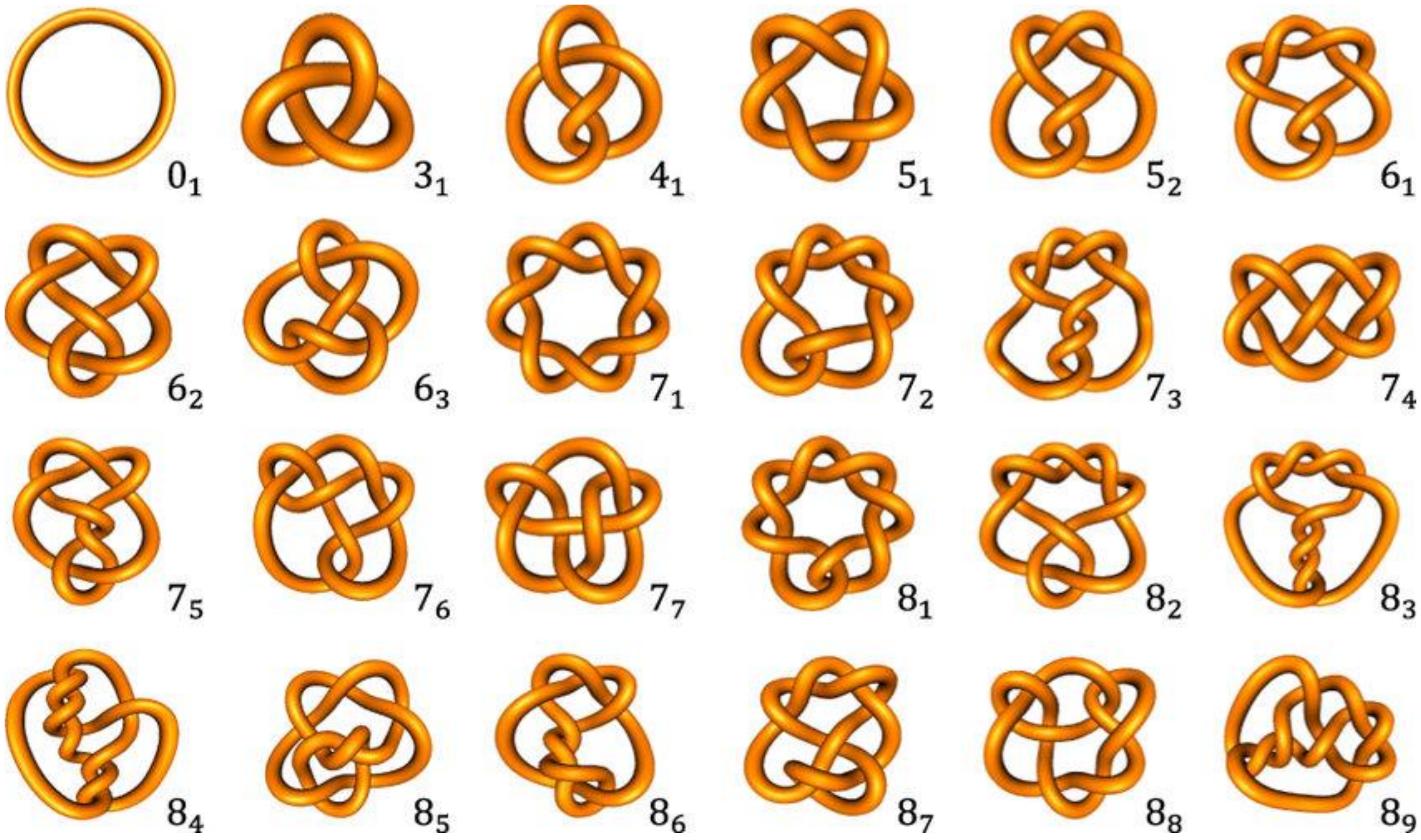
Knot Theory

Or is it?



Definitions

- A Knot K is a smooth embedding

$$K : S^1 \hookrightarrow \mathbb{R}^3$$

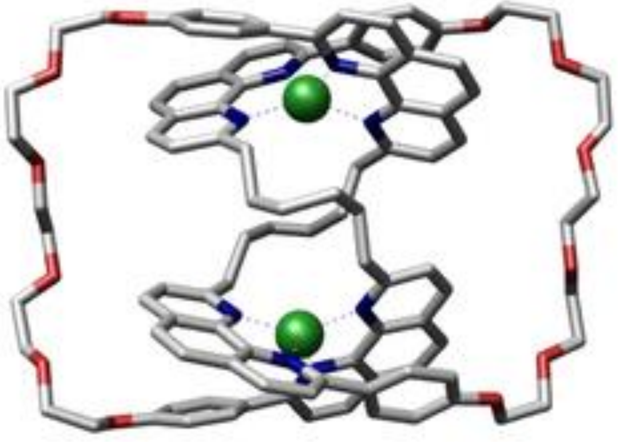
- Two knots are equivalent if there exists an *ambient isotopy* between them

- A projection of a knot is a *regular* projection $\pi : \mathbb{R}^3 \to \mathbb{R}^2$

(limited scenarios where multiple points on the knot are mapped to the same point in the projection)

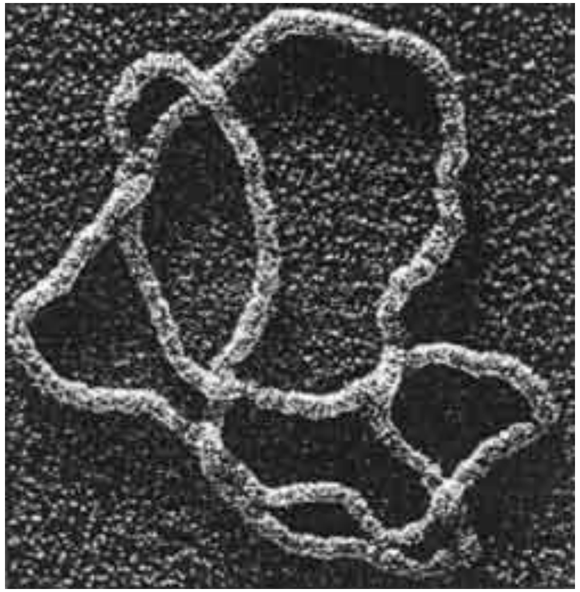
Why Do We Care?

Molecules



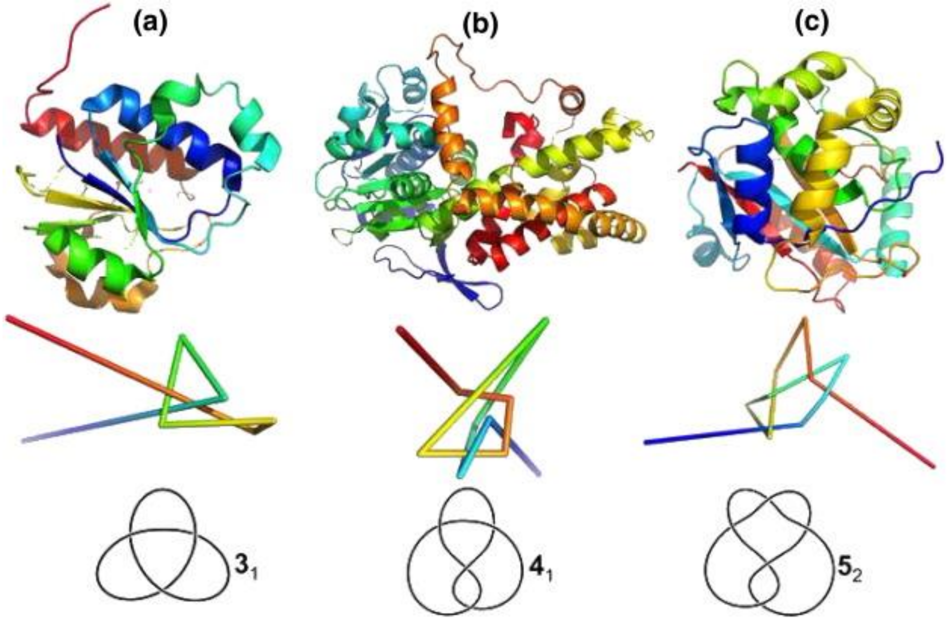
Jean Pierre Sauvage

DNA



Science **229**, 171 (1985);

Proteins



The Classification Problem

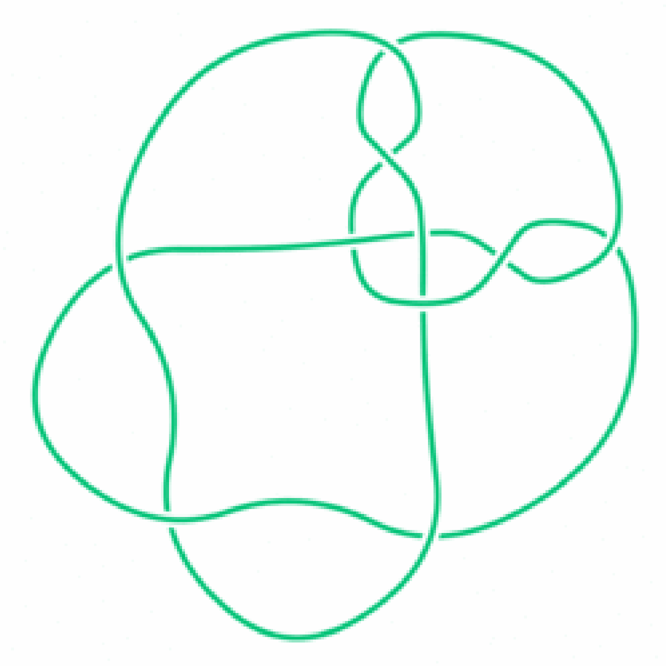
- https://katlas.org/wiki/The_Rolfsen_Knot_Table

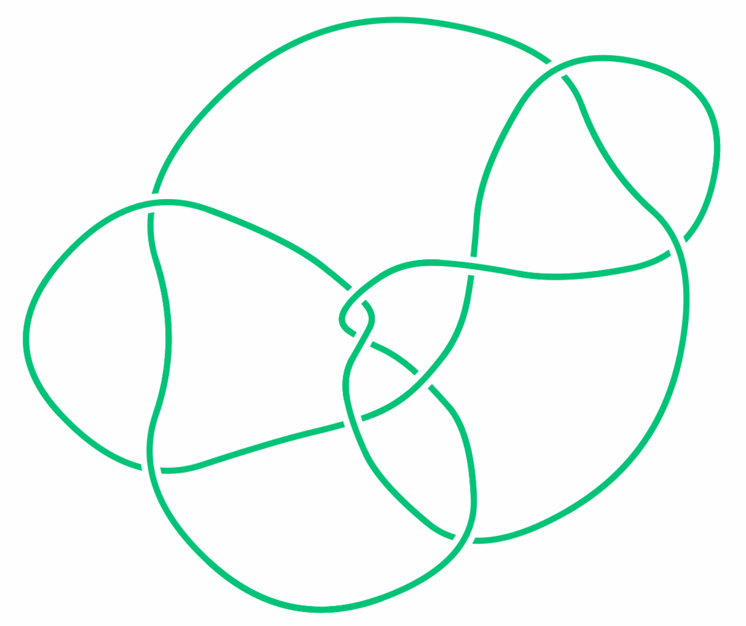
Tabulation Progress

- Gauss
- Peter Guthrie Tait (up to 7 crossings)
- C.N. Little (up to 10 crossings)
- Mary G. Haseman (amphicheiral knots up to 12 crossings)
- John Conway (up to 11 crossings)
- Dowker, Hoste, Thistlethwaite (up to 16)

Number of Crossings	Count of Prime Knots
3	1
4	1
5	2
6	3
7	7
8	21
9	49
10	165
11	552
12	2,176
13	9,988
14	46,972
15	235,293
16	1,388,795

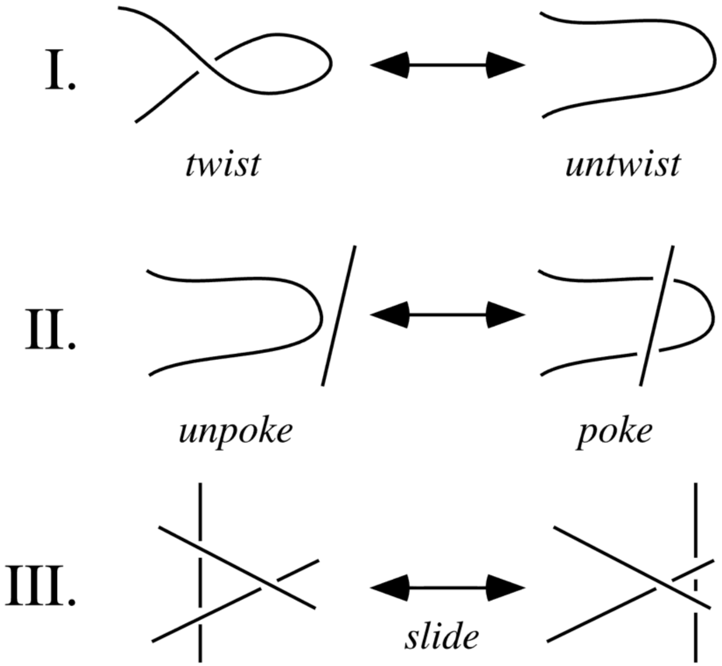
Perko Pair





Reidemeister Moves

Any two projections of a knot can be related by sequences of moves

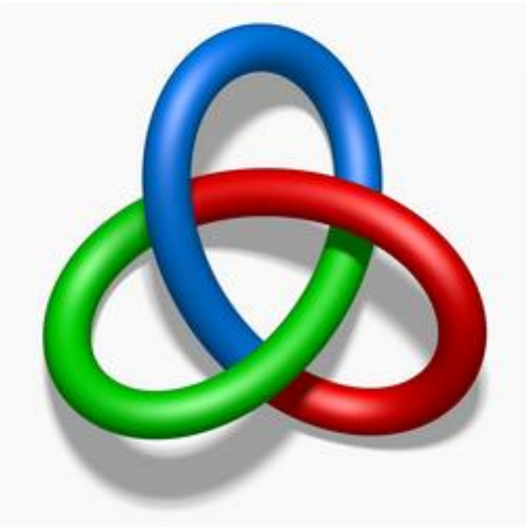


Plus Ambient Isotopy

Tri-color-ability

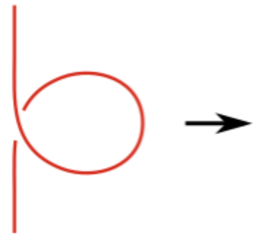
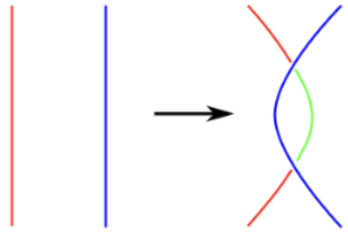
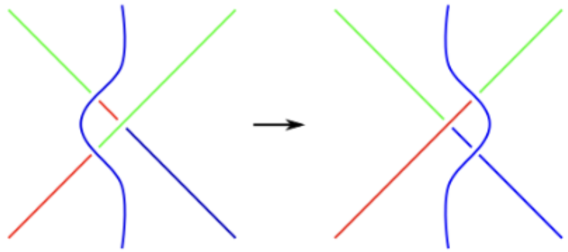
A projection (picture) of a knot is *tricolorable* if each strand can be labeled with one of 3 “colors” such that:

- At least 2 colors are used
- Each intersection is either all the same color or all different.



Tricolorability is a knot invariant

Proof:

Reidemeister Move I is tricolorable.	Reidemeister Move II is tricolorable.	Reidemeister Move III is tricolorable.
		

Other Invariants

- Unknotting Number
- Rope Length
- Bridge number
- Polynomials
- Hyperbolic Volume
- Stick Number
- Knot Group (Fundamental group of knot complement)
- Braid Index
- Tilings

Other Knot Theory Topics

- Surfaces

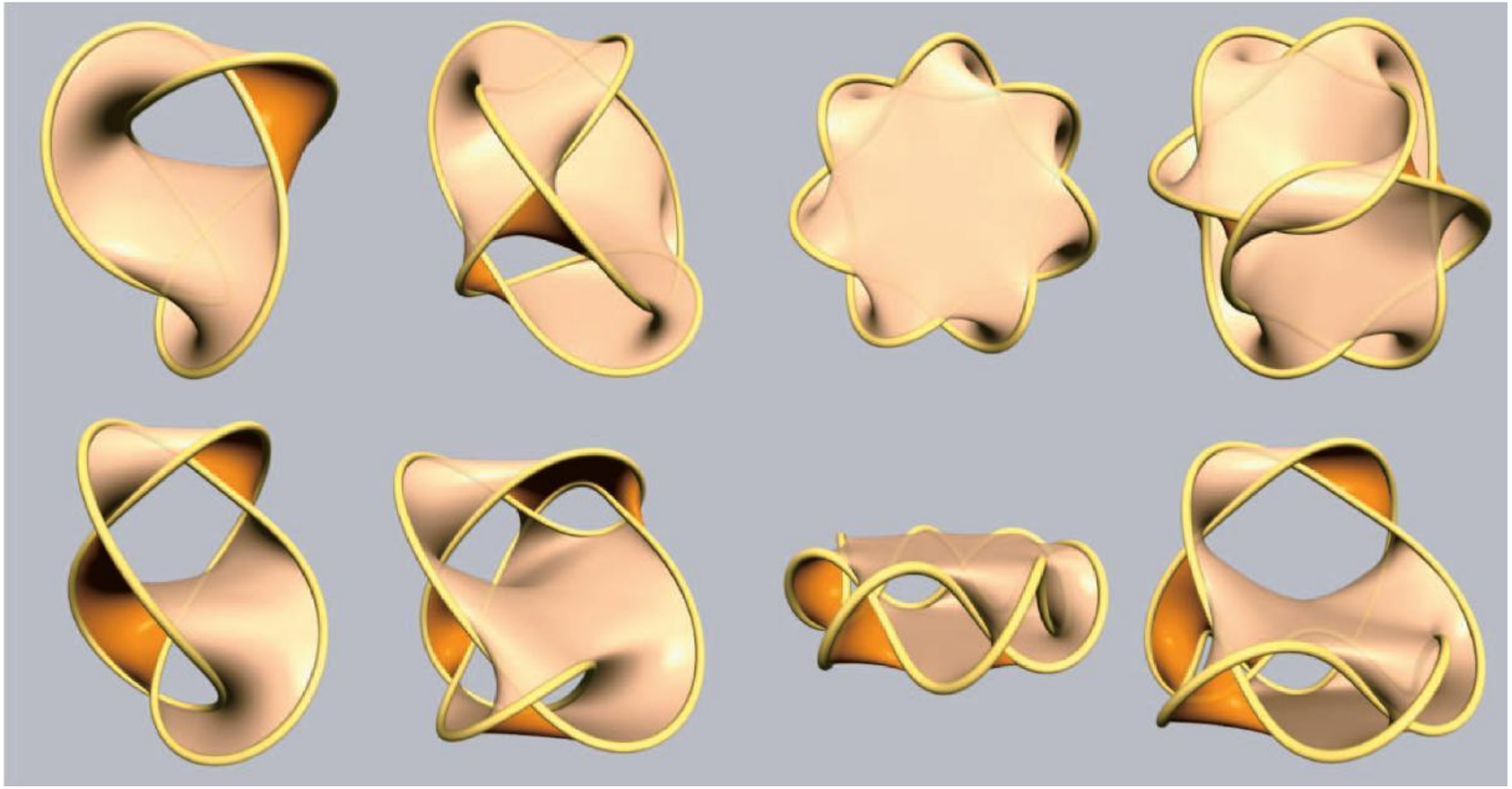


image from Van Wijk, 2006

Sources

- The Knot Book, Colin Adams