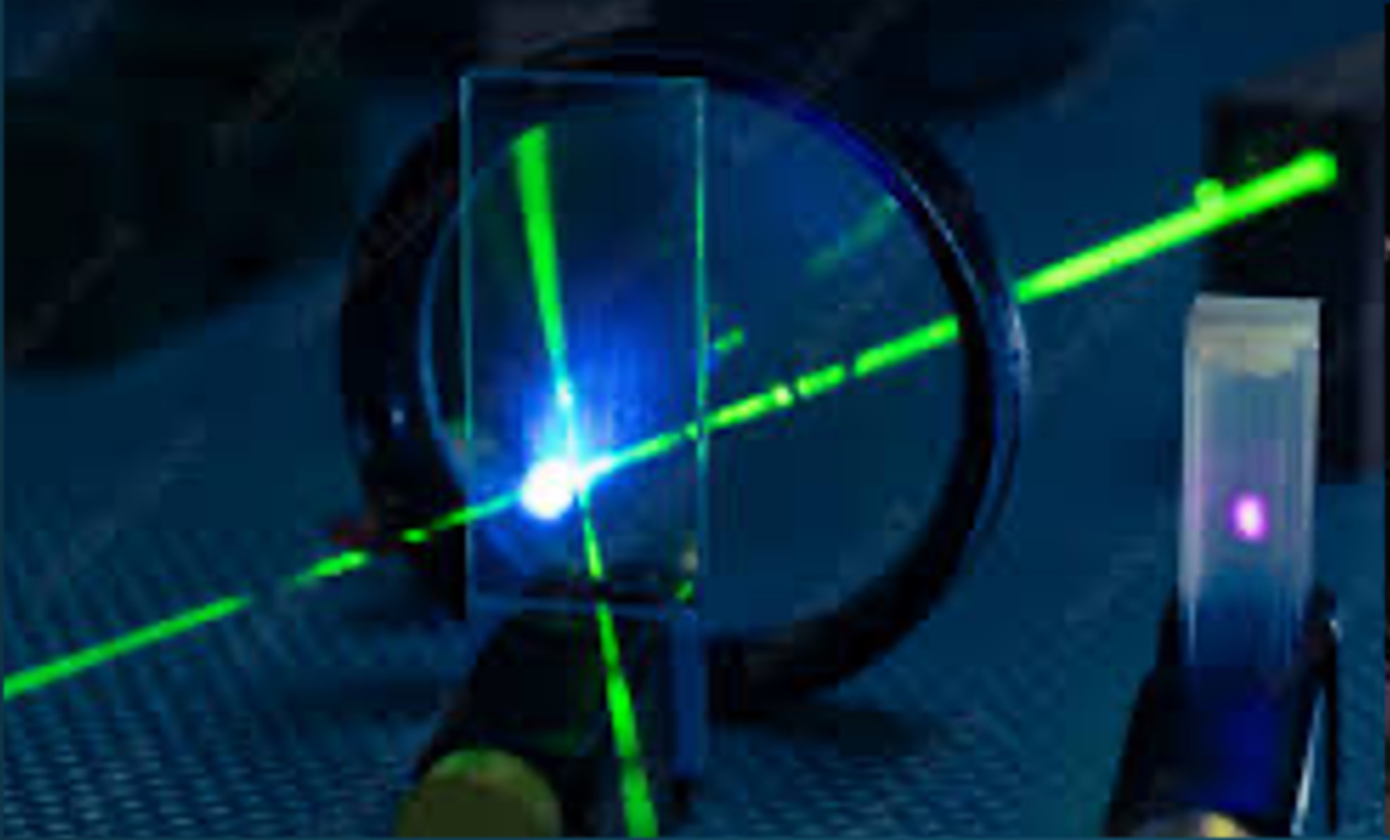



Linear Optical Quantum Computing, AI & The End of The World

Presented by Elijah Burns

Overview:

- Review of theoretical quantum computational logic and Deutsch Oracle
- Basics of LOQC and physical optics
- Quantum operators for both spatially and polarizationally encoded qubits
- Interference in an optical system
- Entanglement in an optical system
- Experimental design for linear optical Deutsch Oracle
- The end of the world as we know it

Key Principles of Quantum Computing:

- Quantum computers use qubits unlike the traditional on-off bits in classical computers.
- Qubits can exist as either a 1 or 0 like a classical bit, but they can also exist in a superposition of the 2 with a certain probability to collapse to 1 and a certain probability to collapse to 0 when measured.
- In addition to superposition, qubits can become entangled making the state of one (or multiple) qubits dependent on that of another and vise versa.
- The last major capability of qubits is their ability to constructively and destructively interfere with one another, allowing us increase the likelihood of desirable results while mitigating unfavorable ones.

Representing Qubits:

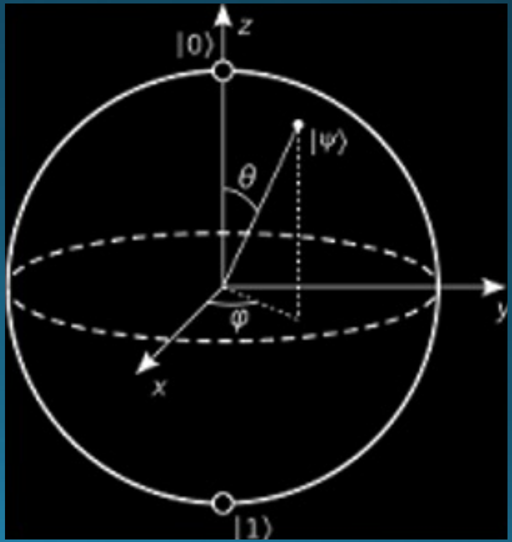
- We represent individual qubits with vectors whose components are the probability coefficients to collapse to 0 ($|0\rangle$) and 1 ($|1\rangle$) respectively:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \mid ||\alpha||^2 + ||\beta||^2 = 1$$

- Systems of multiple qubits are represented by their tensor product:

$$|011\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \begin{pmatrix} 0 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ 1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} \\ 0 \begin{pmatrix} 0 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ 1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



Representing Quantum Logic and Reversible Computing:

- All quantum logic gates can be represented with matrix operations however, all quantum logic gates must be reversible and their own inverse.
- Some of the most important quantum operators:
- The Hadamard gate (H) drops a single qubit in the $|0\rangle$ or $|1\rangle$ state into an equal superposition (50-50 for 1 or 0):

$$H(|0\rangle) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = |+\rangle, \; H(|1\rangle) = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} = |-\rangle$$

Important Quantum Logic Gates Continued:

- Not gate (X) also known as negation flips the probability coefficients of a single qubit:

$$X(|0\rangle) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

- CNot gate (C) is a 2-qubit operation that negates the probability coefficients of a target qubit when the control qubit is 1:

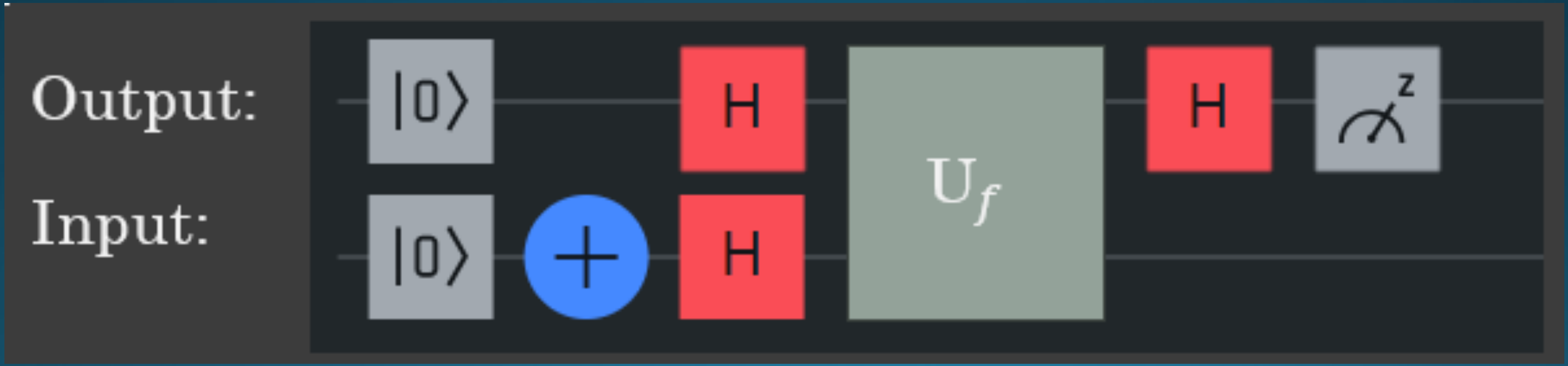
$$C(|+\rangle \otimes |0\rangle) = C\left(\begin{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix}\right) = C\left(\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

- Other gates include: (I won't be going over the matrix representations)
  - phase shifting gates: rotate a qubit around x, y, or z axis by a phase angle.
  - Toffoli gate: CNot gate with 2 controls (quantum AND gate)

Review of Deutsch Oracle:

- The Deutsch Oracle is the simplest case where a quantum computer outperforms a classical computer.
- Checks whether the function in a single bit operator black box is constant or balanced.

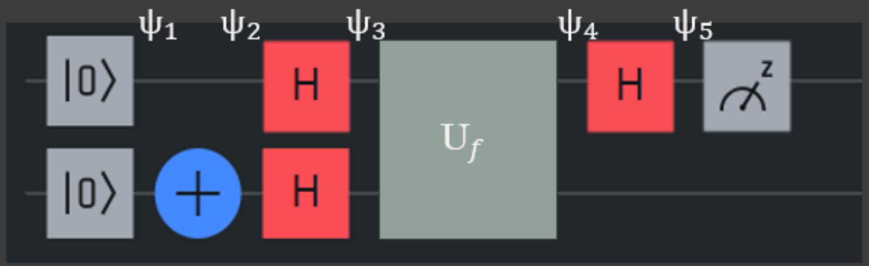
Quantum circuit notation (Robert hates this notation):



Math behind Deutsch Oracle:

Output:

Input:



$$|\psi_1\rangle = |00\rangle \Rightarrow |\psi_2\rangle = |01\rangle$$

$|\psi_3\rangle = |+-\rangle$ ($|+\rangle$ represents $H(|0\rangle)$ while $|-\rangle$ represents $H(|1\rangle)$)

$$|\psi_3\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\,|-\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle|-\rangle + |1\rangle|-\rangle)$$

$$|\psi_4\rangle = U(|\psi_3\rangle) = \tfrac{1}{\sqrt{2}}(U(|0\rangle)|-\rangle + U(|1\rangle)|-\rangle)$$

$$|\psi_4\rangle = U(|\psi_3\rangle) = \tfrac{1}{\sqrt{2}}((-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle)|-\rangle$$

- Considering our 2 possible cases; Constant: $f(0) = f(1)$, Balanced: $f(0) \neq f(1)$:

$$(-1)^{f(0)} = (-1)^{f(1)} = \pm 1 \Rightarrow |\psi_4\rangle = \pm\tfrac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \pm|+\rangle$$

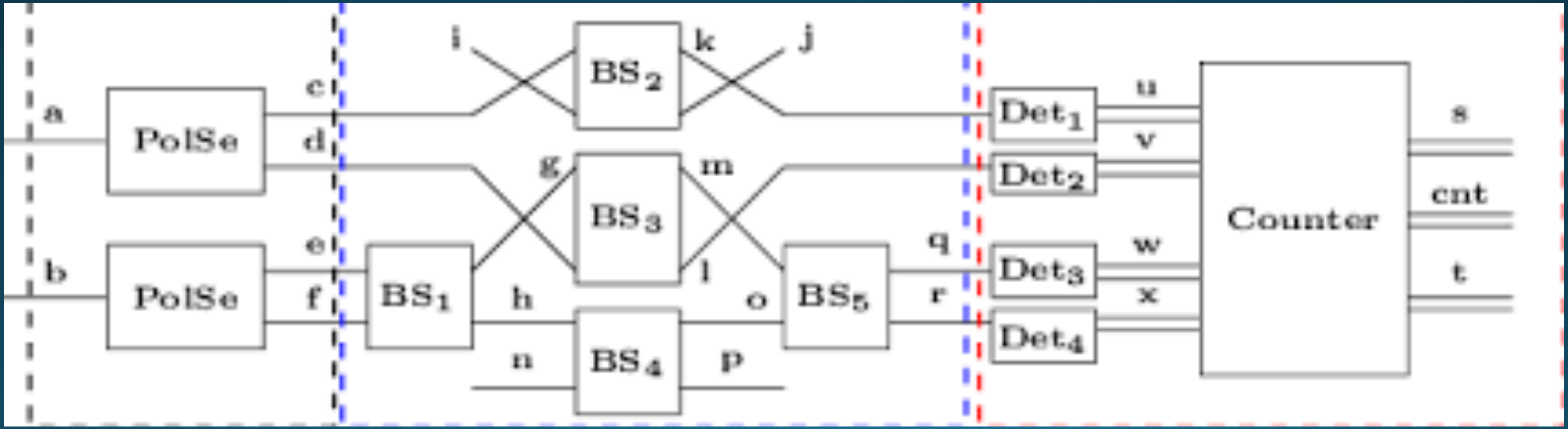
$$(-1)^{f(0)} = \pm 1 = -(-1)^{f(1)} = -(\mp 1) \Rightarrow |\psi_4\rangle = \pm\tfrac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \pm|-\rangle$$

Constant: $|\psi_5\rangle = \pm H(|\psi_4\rangle) = \pm H(|+\rangle) = \pm|0\rangle$

Balanced: $|\psi_5\rangle = \pm H(|\psi_4\rangle) = \pm H(|-\rangle) = \pm|1\rangle$

General overview of Linear Optical Quantum Computing:

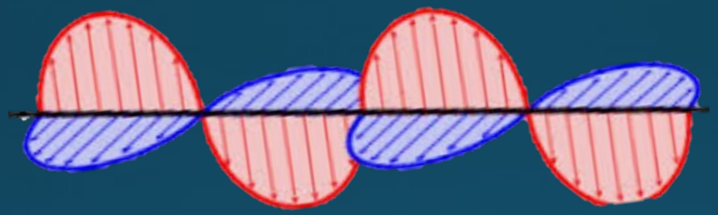
- LOQC uses lasers (more specifically the photons in the beams) as qubits placing different optical equipment in the linear path of the beam to enact quantum operators before measuring the output.



Representing light as EM wave function:

- EM waves (light) can be represented with the standard wave equation:

$$\Psi(x,t) = (E_m \sin(kx \pm \omega t + \phi))\hat{j} + (B_m \sin(kx \pm \omega t + \phi))\hat{k}$$



- The direction of propagation (+x or $\hat{i}$) is defined by the Poynting vector:

$$\vec{S} \propto \vec{E} \times \vec{B} \; (\vec{E} \text{ and } \vec{B} \; are\ always\ perpendicular)$$

- Because E and B are always perpendicular and proportional we don't actually need to look at both, simplifying our function to the following:

$$\Psi(x,t) = A\sin(kx \pm \omega t + \phi)$$

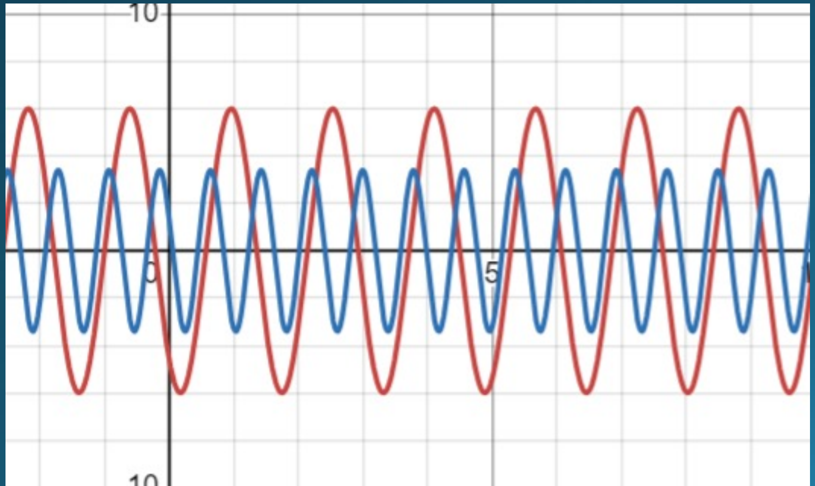
Representing EM waves with complex exponentials:

- Using Euler's Formula, we can rewrite this as the complex component of a complex exponential:

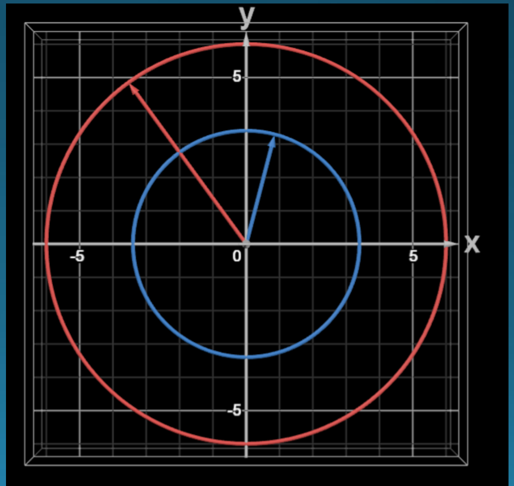
$$\Psi(x,t) = \mathrm{Im}(A\cos(kx \pm \omega t+\phi) + iA\sin(kx \pm \omega t +\phi))$$

$$\Psi(x,t) = \mathrm{Im}(Ae^{i(kx\pm\omega t+\phi)})$$

- This takes us from trig functions in the $\mathbb{R}^2$ to vectors on circles in the complex plane:

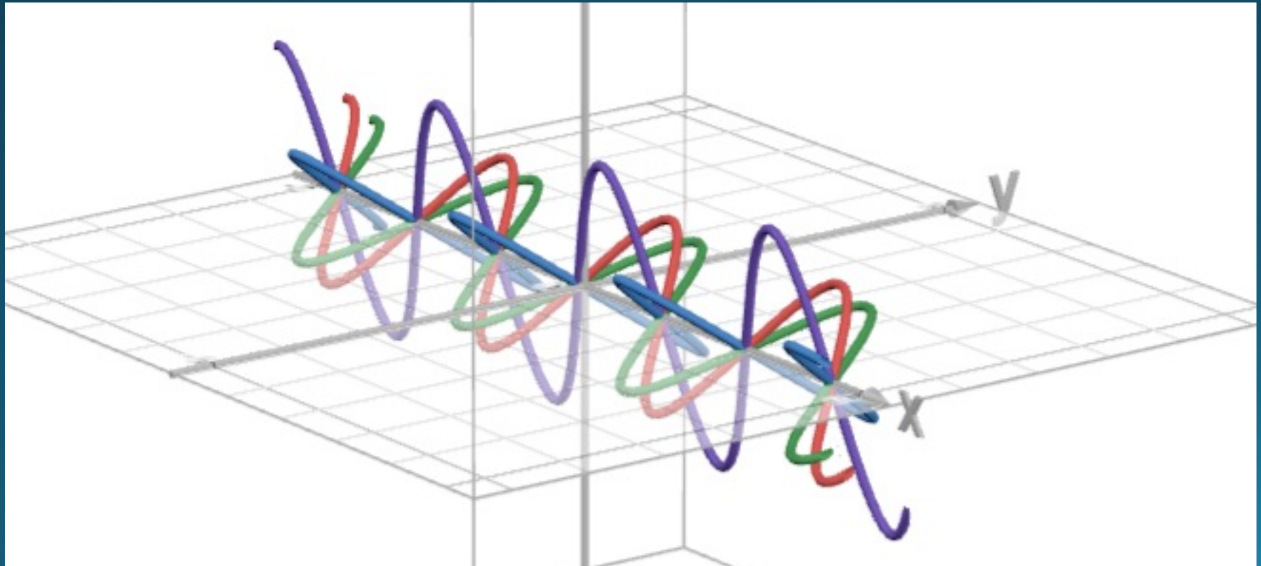


$\Rightarrow$



Polarization of light:

- EM waves also have a polarization angle (rotation around the axis of propagation).
- Light from a laser beam has a uniform wavelength however depending on the type may produce unpolarized light:
- Largely these can all be broken up into vertical and horizontal components which we call s and p polarized respectively:



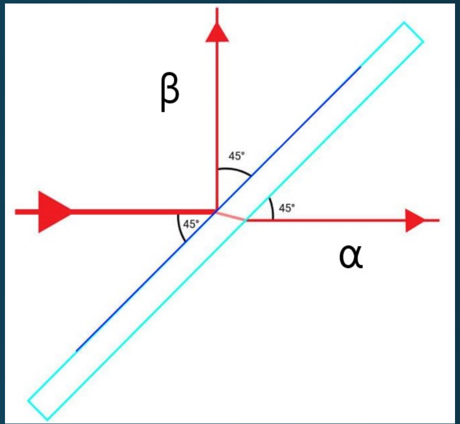
Spatial encoding of photonic qubits:

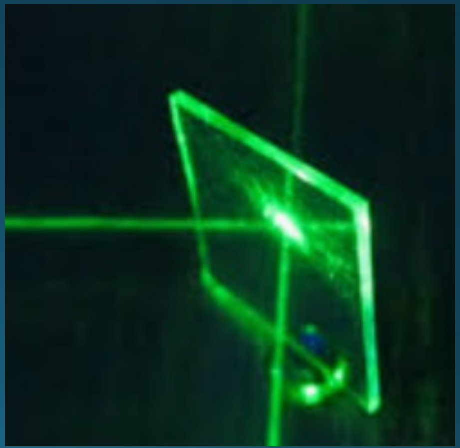
- When specially encoding a beam, we represent photonic qubits with beam pairs, each corresponding to a probability coefficient of the $|0\rangle$ and $|1\rangle$ states respectively:
- Given this, the $|0\rangle$ is just a singular beam which can be placed into equal superposition through the use of a traditional beam splitter (50-50, non-polarizing).
- The matrix operator of a 50-50, lossless, dielectric beam splitter is equivalent to the theoretical Hadamard:

$$\mathrm{H}(|0\rangle) \cong Bs\begin{pmatrix}\Psi(x_b,t)\\ 0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}\Psi(x_b,t)\\ 0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\Psi(x_b,t)\\ \Psi(x_b,t)\end{pmatrix}$$

- When considering the entire beam or pair of beams as a photonic qubit the following relationship must hold:

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix} \;\Big|\; \alpha = \sqrt{\frac{\Psi_0(x,t)}{\Psi_0(x,t)+\Psi_1(x,t)}},\ \beta = \sqrt{\frac{\Psi_1(x,t)}{\Psi_0(x,t)+\Psi_1(x,t)}}$$





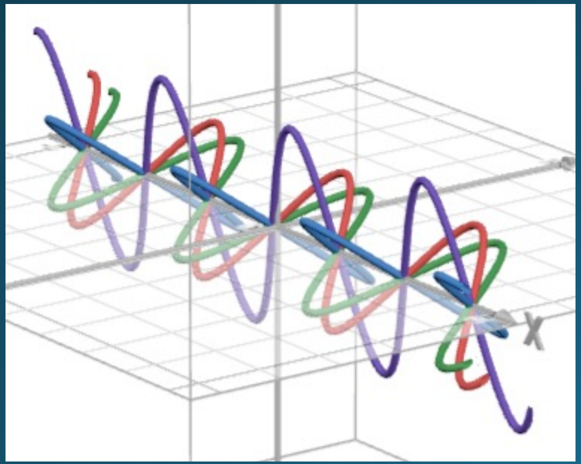
Polarization encoding:

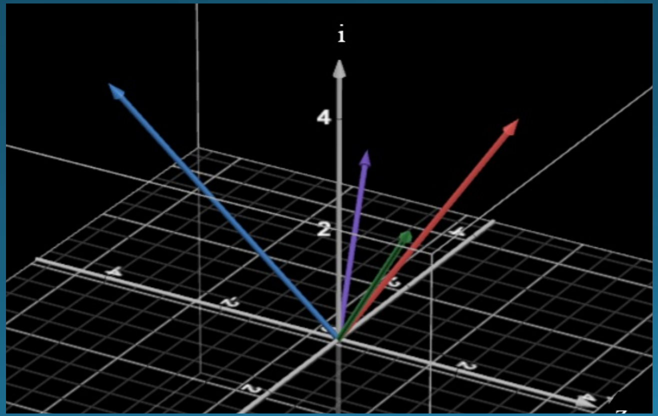
- Another way we can encode photonic qubits is in the polarization of their wave function.

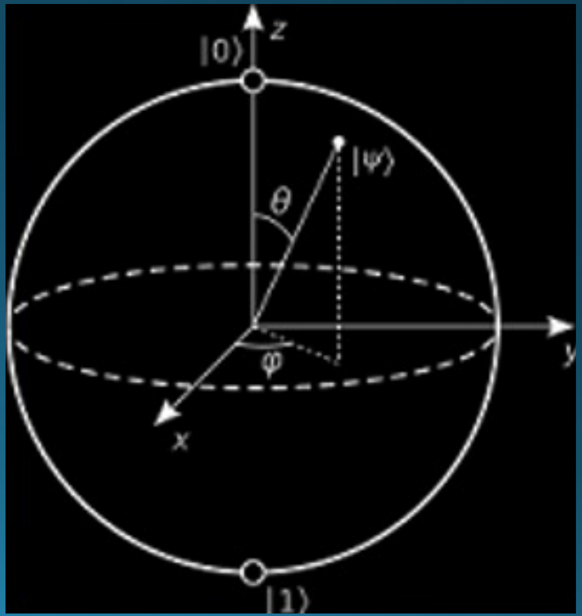
$$\Psi(x,t) = \begin{pmatrix} \mathrm{Im}(A_p e^{i(kx \pm \omega t + \phi)}) \\ \mathrm{Im}(A_s e^{i(kx \pm \omega t + \phi)}) \end{pmatrix} = \begin{pmatrix} \Psi_p(x,t) \\ \Psi_s(x,t) \end{pmatrix} \cong \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

- Under this representation the $|0\rangle$ and $|1\rangle$ states are represented as follows:

$$|0\rangle \cong \begin{pmatrix} \Psi_p(x,t) \\ 0 \end{pmatrix}, |1\rangle \cong \begin{pmatrix} 0 \\ \Psi_s(x,t) \end{pmatrix}$$





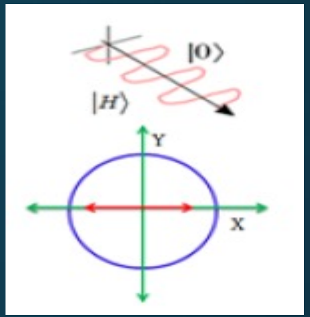


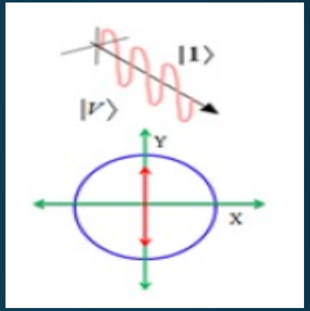
Superposition of polarization encoded with wave plates:

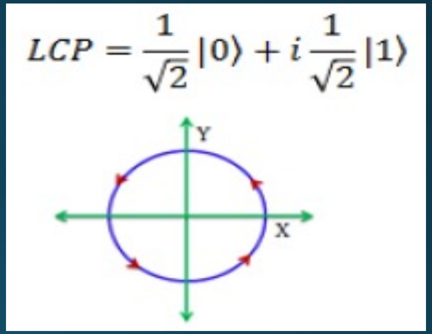
- To place a polarization encoded beam into superposition, a phase shift must be enacted.
- Most commonly this is through the use of a quarter wave plate which enacts a $\frac{\pi}{4}$ phase shift:

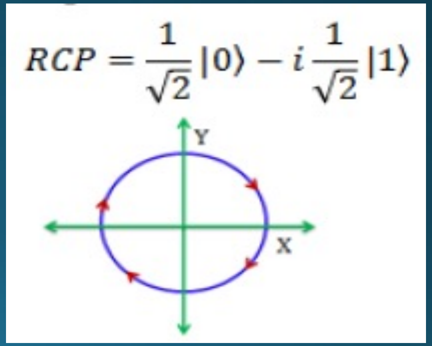
$$qwp(|0\rangle) = \frac{1}{2\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\begin{pmatrix} \Psi(x,t) \\ 0 \end{pmatrix} = \frac{1}{2\sqrt{2}}\begin{pmatrix} \Psi(x,t) \\ i\Psi(x,t) \end{pmatrix}$$

$$qwp(|1\rangle) = \frac{1}{2\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\begin{pmatrix} 0 \\ \Psi(x,t) \end{pmatrix} = \frac{1}{2\sqrt{2}}\begin{pmatrix} \Psi(x,t) \\ -i\Psi(x,t) \end{pmatrix}$$



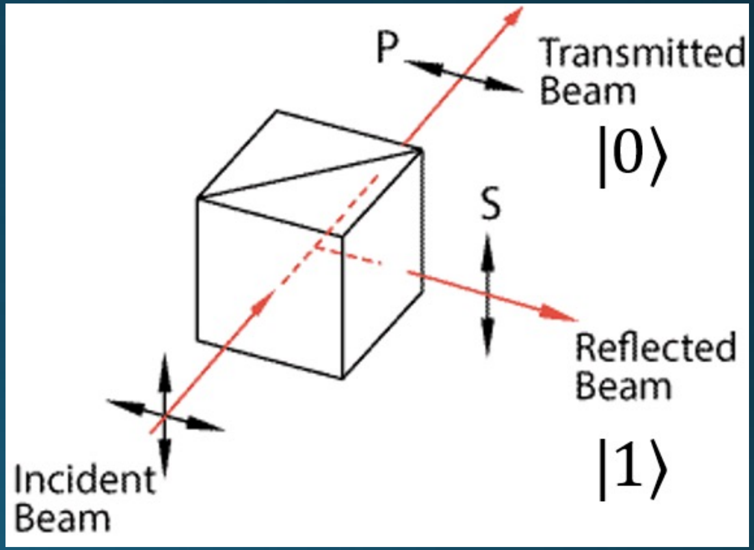




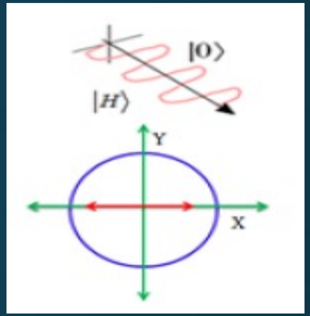


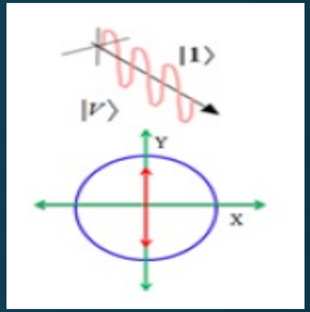
Converting from Polarization to Spatial encoding and vise versa:

- When an unpolarized beam is passed through a polarized beam splitter, the s polarized component is reflected while the p polarized component is transmitted.
- In practice this takes a beam comprised of 2 polarization encoded qubit states and separates them into 2 paths (in this case, each path is its own qubit rather than being components of the same qubit as with spatial encoding).
- A polarized beam splitter can also be used to convert from spatial encoding to polarization encoding and vise versa.



Polarization and spatial operators:



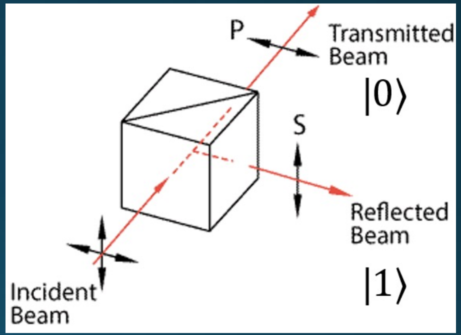


- The general form matrix operator for a beam splitter is dependent on reflection and transmission coefficients r and t ($r^2 + t^2 = 1$, r = sin θ, t = cos θ):

$$e^{i\phi_0}\begin{pmatrix} re^{i\phi_t} & te^{-i\phi_t} \\ te^{i\phi_t} & -re^{-i\phi_r} \end{pmatrix}$$

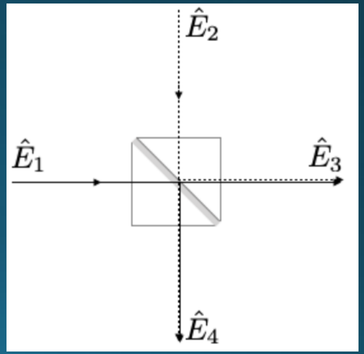
- In the symmetric case (such as a dielectric non-polarizing beam splitter) when ϕ_0 = ϕ_t = ϕ_r= 0 and θ = $\frac{\pi}{4}$ we essentially get the theoretical Hadamard:

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

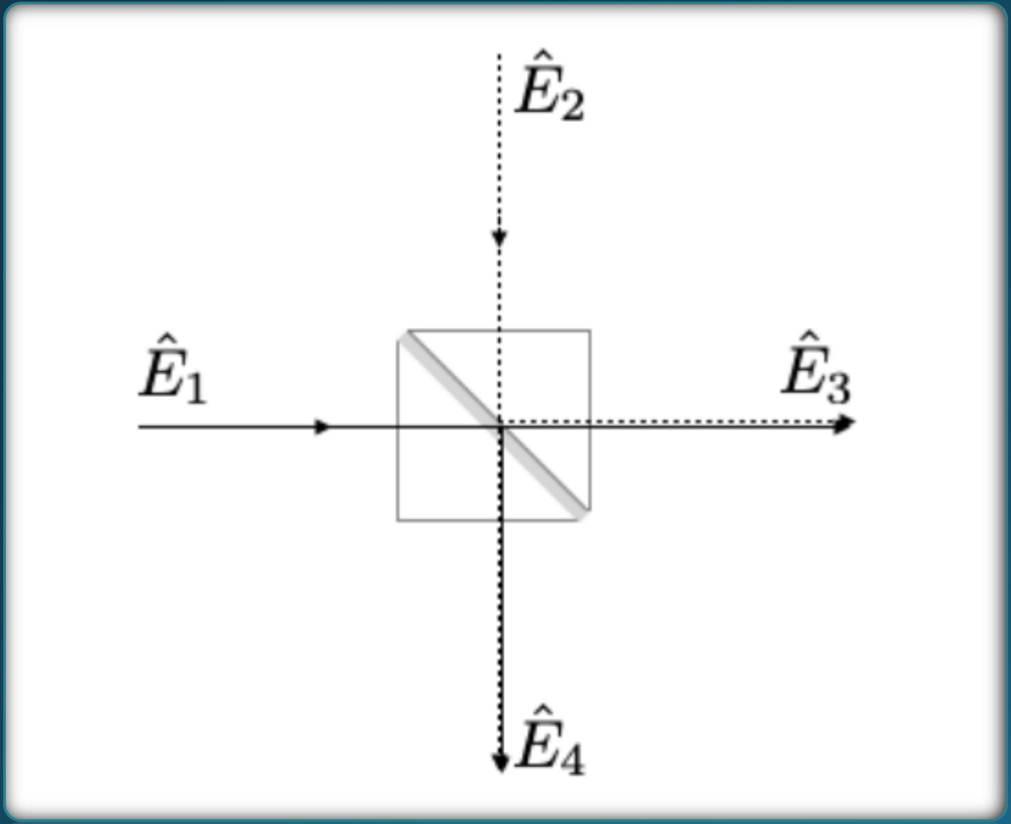


- With a PBS a phase shift is often enacted on the reflected beam. In practice ϕ_r and ϕ_0 are often π, so if ϕ_t = 0 and θ = $\frac{\pi}{4}$ we get the following transformation ($e^{i\pi} = -1$):

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$



Interference of photonic qubits:

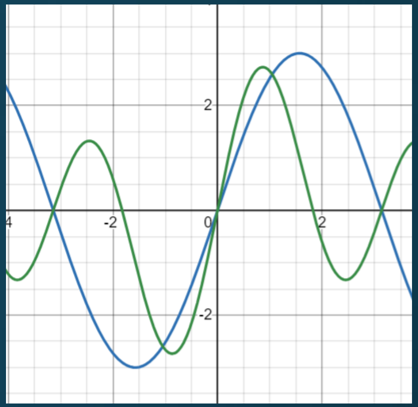


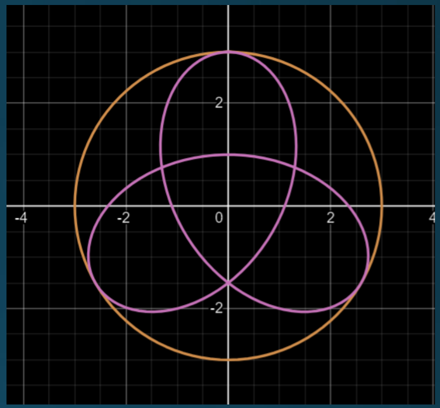
- In practice, Beam splitters can have 2 inputs, and those 2 inputs can interact when properly aligned.
- This allows the wave functions of each input beam to interact and interfere with each other.
- For this to happen predictively we need our 2 input beams to be coherent, which basically means that they have very similar if not the same wavelength.
- Using a single laser (that is split multiple times) as the source for all qubits in a circuit is often how coherence is achieved.

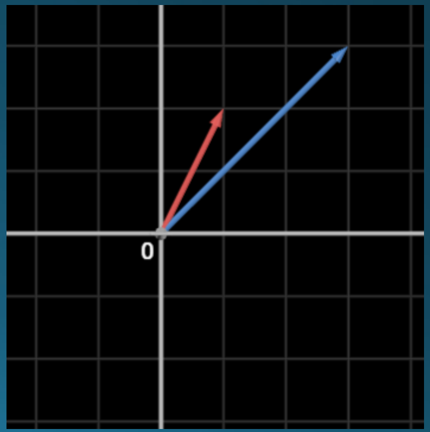
Necessity of Coherence to interfere qubits:

- Intuitively, when adding 2 sin functions of the same wavelength you will get out another sin function of that wavelength (the amplitude and phase might differ, but wavelength is preserved).
- This becomes apparent in the factorability of the complex exponentials:

$$A_1 e^{i(\omega t + \phi_1)} + A_2 e^{i(\omega t + \phi_2)}$$
$$= e^{i(\omega t)}\left(A_1 e^{i(\phi_1)} + A_2 e^{i(\phi_2)}\right)$$

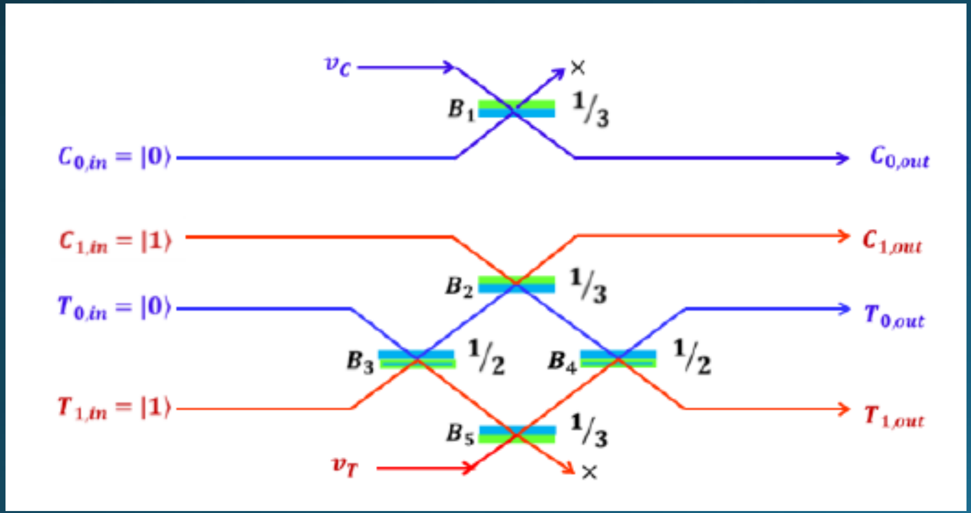


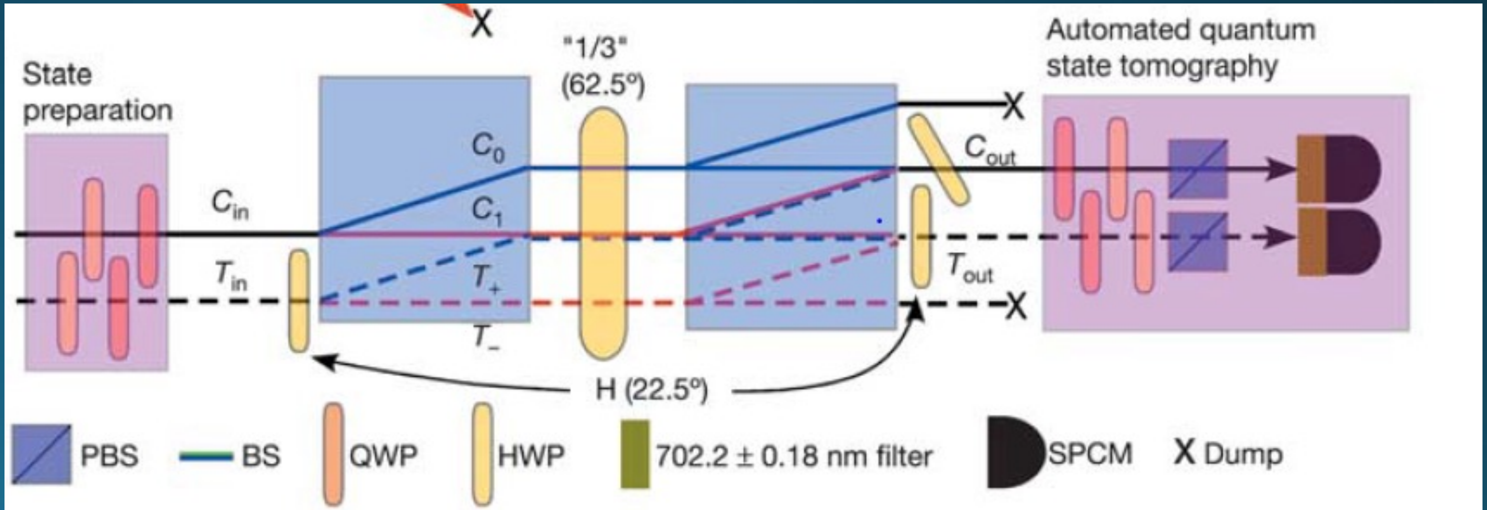




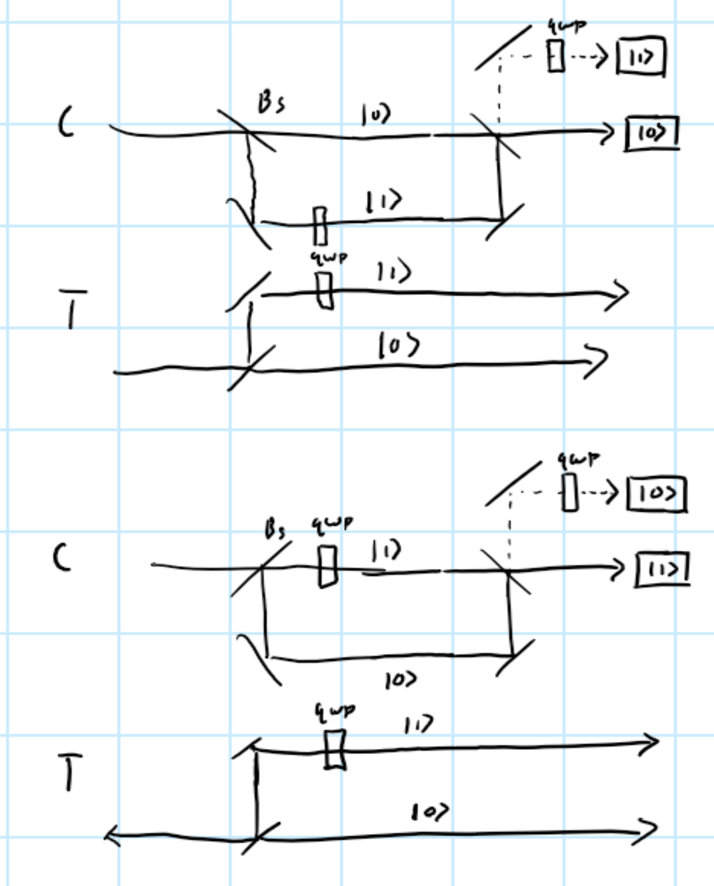
Entanglement with Optical CNot:

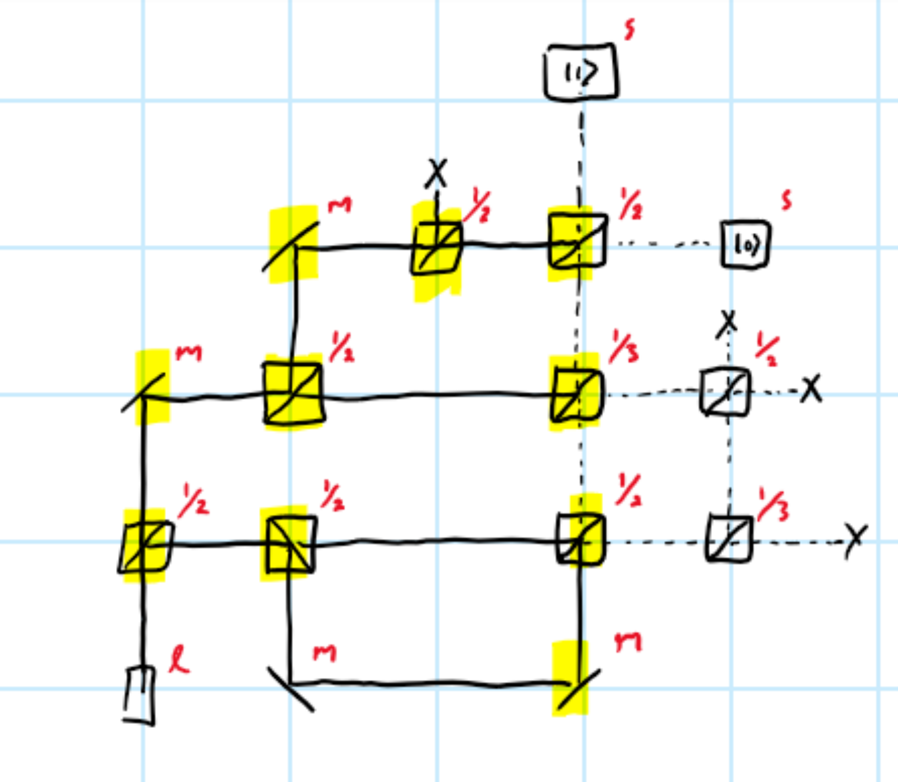
- Achieving entanglement in practice is very difficult no matter what kind of quantum computer you are dealing with.
- With photonic qubits it requires careful interference of 2 specially encoded beams.
- After reading multiple papers on the subject this seems to be the most robust and widely accepted implementation of an optical CNot.

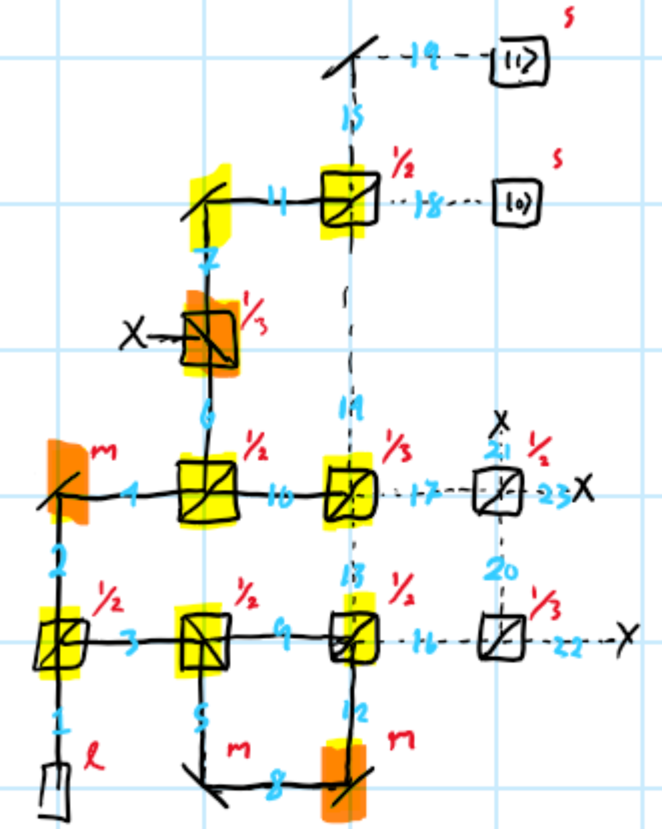




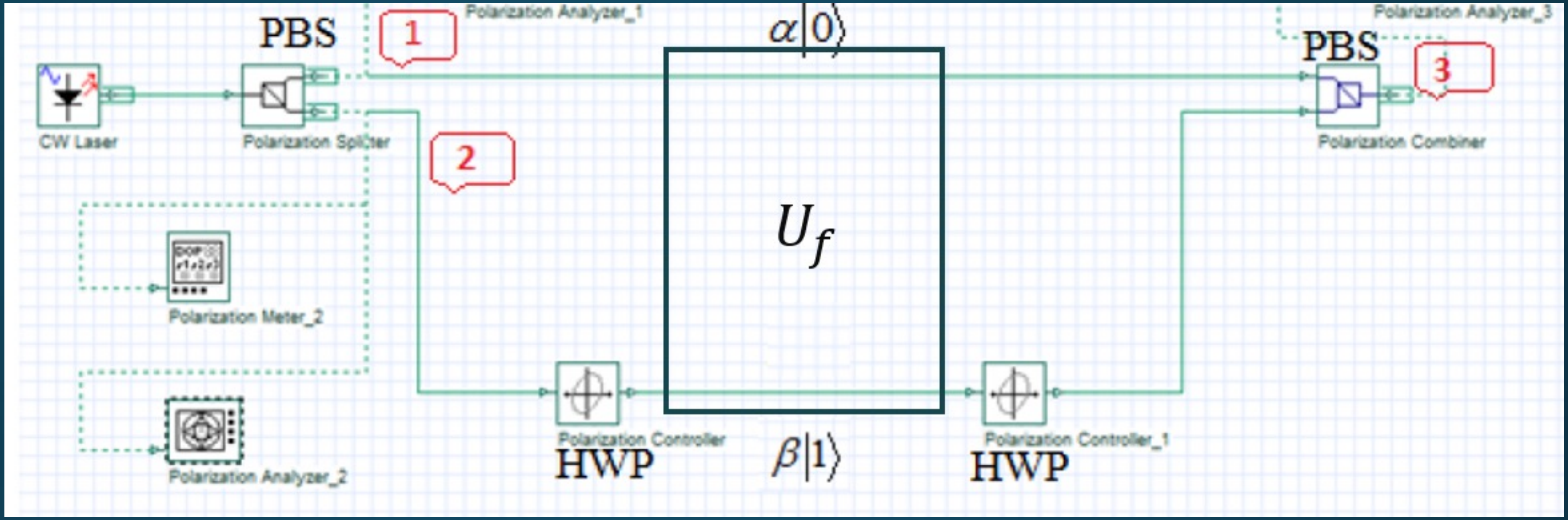
Optical Deutsch Oracle design iterations:

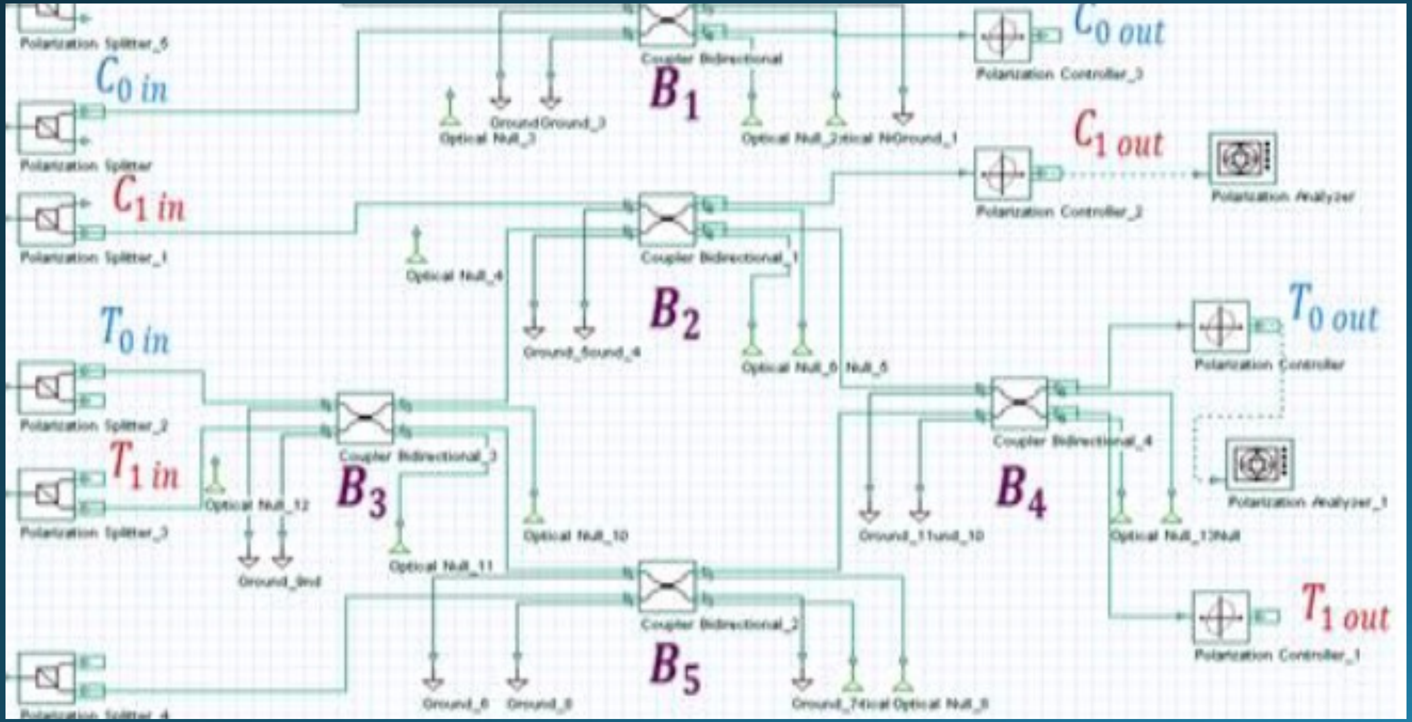






Final design:





Balanced case oracle function:

Basics of Neural Networks:

- Artificial Intelligence, from language models to its most basic forms can largely be boiled down to neural networks, which in turn can largely be boiled down to long series of matrix operators.

$$\begin{bmatrix} x_{i1} \\ \vdots \\ x_{in} \end{bmatrix} \Rightarrow M_n(\ldots(M_1(\boldsymbol{x}))) \Rightarrow \begin{bmatrix} x_{o1} \\ \vdots \\ x_{on} \end{bmatrix} \Rightarrow \begin{bmatrix} x_{e1} \\ \vdots \\ x_{en} \end{bmatrix}$$

4 pillars of computational intelligence:

- Knowledge (K)
- Memory (M)
- Computational speed (S)
- Creativity/Problem solving (P)