Hilbert Space Methods Used in a First Course in Quantum Mechanics:

The Aharonov-Bohm Effect

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06/05/14

History of the Aharonov-Bohm Effect

1949. Werner Ehrenberg and Raymond E. Siday first predicted the effect in semi-classical form. [**Werner Ehrenberg** (1901-1975) was a English physicist. **Raymond E. Siday** (1912–1956) was an English mathematician specializing in quantum mechanics.]

Y. Aharonov (born in 1932)

David Bohm (1917 – 1992)

1959. Yakir Aharonov and David Bohm published their seminal paper with a detailed description and explanation of this and a number of related phenomena. [**Yakir Aharonov** is an Israeli physicist specializing in quantum physics. **David Joseph Bohm** was an American theoretical physicist.] After publication of the 1959 paper, Bohm was informed of Ehrenberg and Siday's work, which was acknowledged and credited in Bohm and Aharonov's subsequent 1961 paper.

1986. The existence of the effect is no longer in doubt thanks to conclusive experiments of Akira Tonomura.

Akira Tonomura
(1942 – 2012)

The correct name should be "The Ehrenberg-Siday-Aharonov-Bohm Effect"

The vector potential A

- **The vector potential A:** It is a vector field defined along with the electric potential ϕ (a scalar field) by the equations:

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}$$

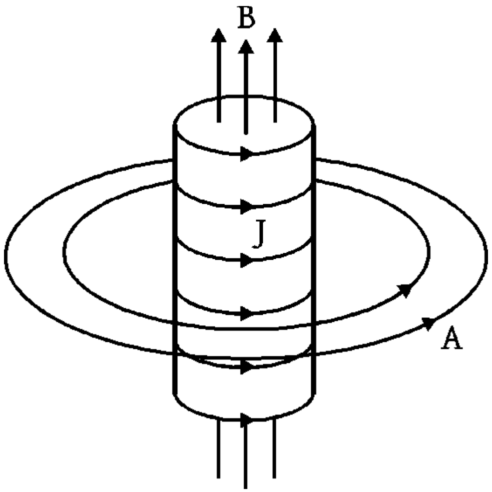
- **Gauge freedom (The Lorenz gauge condition):** In an inertial frame of reference, the vectors **E** and **B** do not change if we take any function *f*(**r**, *t*) and simultaneously transform **A** and *f* as follows:

$$\phi \to \phi - \frac{\partial f}{\partial t}, \quad \mathbf{A} \to \mathbf{A} + \nabla f$$

Quantum Mechanics is Invariant with Respect to the Gauge Calibration

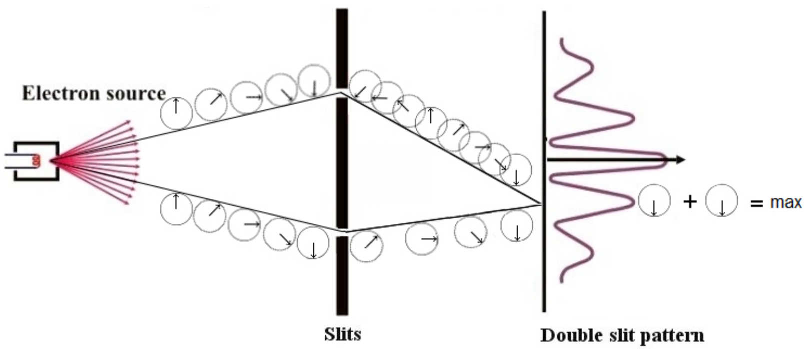
Lorenz force: $\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$

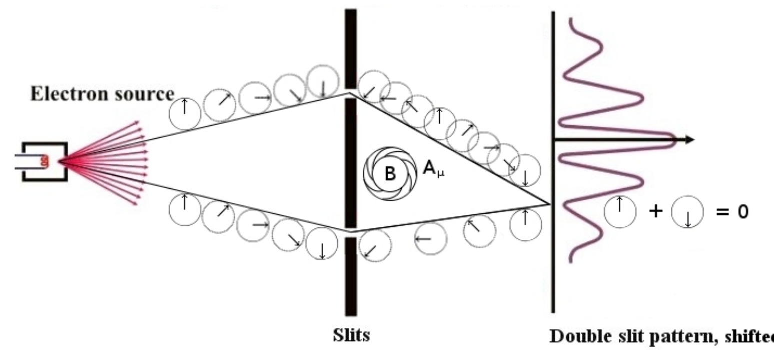
$$\varphi = \frac{q}{\hbar}\oint \mathbf{A} \cdot d\mathbf{r}$$



Tonomura's Experiment (1986):

Double-Slit Diffraction of Electrons.





$$H = \frac{1}{2m}(\mathbf{p} - e\mathbf{A})^2 - \mu\boldsymbol{\sigma}\cdot\mathbf{B}, \qquad \mathbf{p} = \langle p_x, p_y, 0\rangle$$

$$\mathbf{A} = \frac{\Phi}{2\pi}\frac{\mathbf{e}_z \times \mathbf{r}}{\rho^2}, \qquad \psi = e^{iM\varphi}P(\rho)$$

$$P'' + \frac{1}{\rho}P' + \left(k^2 - \frac{(M-\lambda)^2}{\rho^2}\right)P = 0, \quad \lambda = \frac{\Phi}{\Phi_0}, \quad \Phi_0 = \frac{2\pi\mathrm{h}c}{e}$$

Akira Tonomura
(1942 – 2012)