Tying Knots in Electromagnetic Field

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Tying Knots in Light Fields

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We construct analytically, a new family of null solutions to Maxwell's equations in free space whose field lines encode all torus knots and links. The evolution of these null fields, analogous to a compressible flow along the Poynting vector that is shear free, preserves the topology of the knots and links. Our approach combines the construction of null fields with complex polynomials on $\mathbb{S}^3$. We examine and illustrate the geometry and evolution of the solutions, making manifest the structure of nested knotted tori filled by the field lines.

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Knots and the application of mathematical knot theory to space-filling fields are enriching our understanding of a variety of physical phenomena with examples in fluid dynamics [1–3], statistical mechanics [4], and quantum field theory [5], to cite a few. Knotted structures embedded in physical fields, previously only imagined in theoretical

remarkable structure known as a Hopf fibration, with each field line forming a closed loop such that any two loops are linked. At time $t = 0$, each of the electric, magnetic, and Poynting field lines have identical structure (that of a Hopf fibration), oriented in space so that they are mutually orthogonal to each other. The topology of these structures

Is a Photon a Wave Packet?

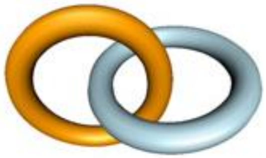
- A wave packet can be analyzed into, or can be synthesized from, an infinite set of component sinusoidal waves of different wavenumbers, with phases and amplitudes such that they interfere constructively only over a small region of space, and destructively elsewhere.
- Equation of a traveling plane wave: $y(x,t) = Ae^{i(kx-\omega t+\varphi_0)}$
- For a wave packet,

$$y(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(k,\omega)e^{i(kx-\omega t)}dk\,d\omega$$

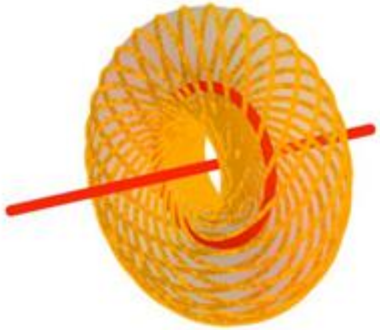
$$|\Delta k| \approx \frac{2\pi}{|\Delta x|} \quad \Rightarrow \quad |\Delta x||\Delta k| \approx 2\pi, \qquad |\Delta t| \approx \frac{2\pi}{|\Delta \omega|} \quad \Rightarrow \quad |\Delta \omega||\Delta \omega| \approx 2\pi$$

- Schrödinger suggested Gaussian wave packets.
- A Gaussian State Moving at constant momentum
- In one-dimension, a soliton-like excitation (a wave packet) dissipates with time. To avoid dissipation, we have to assume a non-linear wave equation.

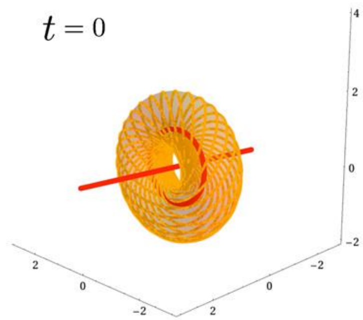
Wave Packets in 3D (W.T.M. Irvine, 2008)

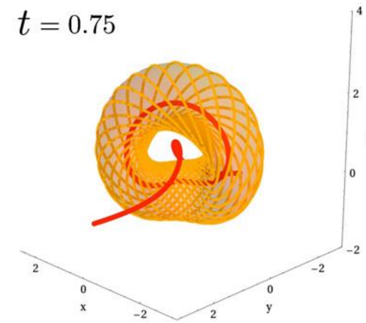
- Combined electric and Magnetic fields: 

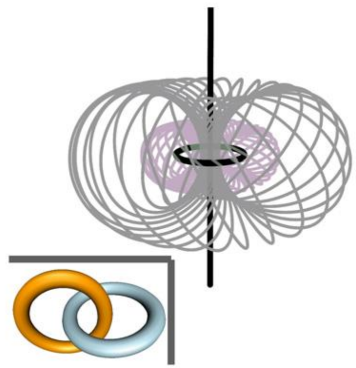
- Toroidal solution of W.T.M Irvine (2008):

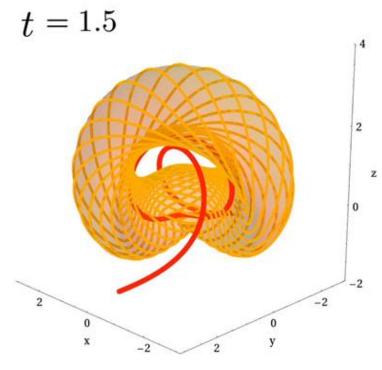


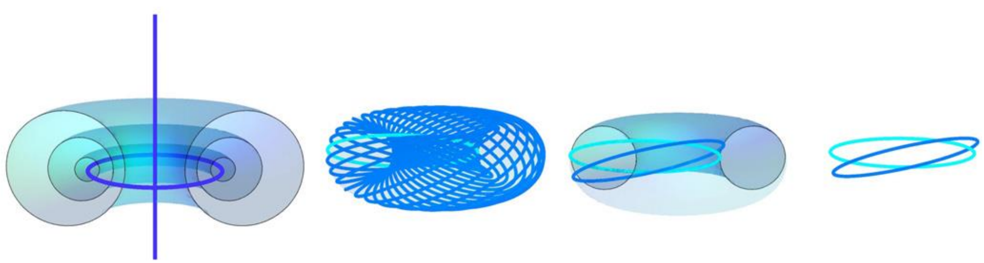
Toroidal solution evolves into a Hopf fibration:





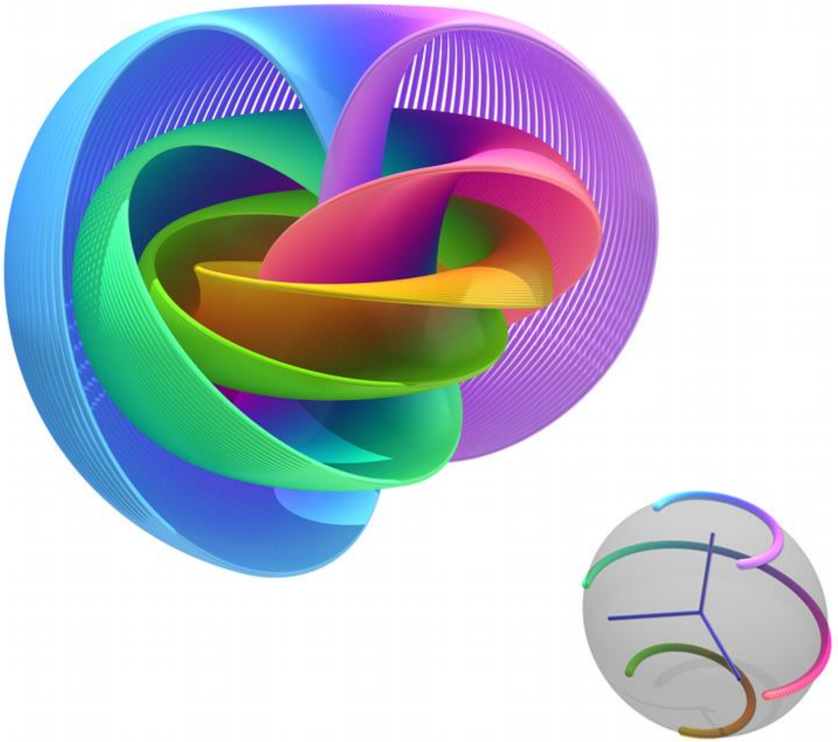






Hopf Fibrations

In topology, the **Hopf fibration** (also known as the **Hopf bundle** or **Hopf map**) describes a 3-sphere (a hypersphere in four-dimensional space) in terms of circles and an ordinary sphere. Discovered by Heinz Hopf in 1931, it is an influential early example of a fiber bundle.

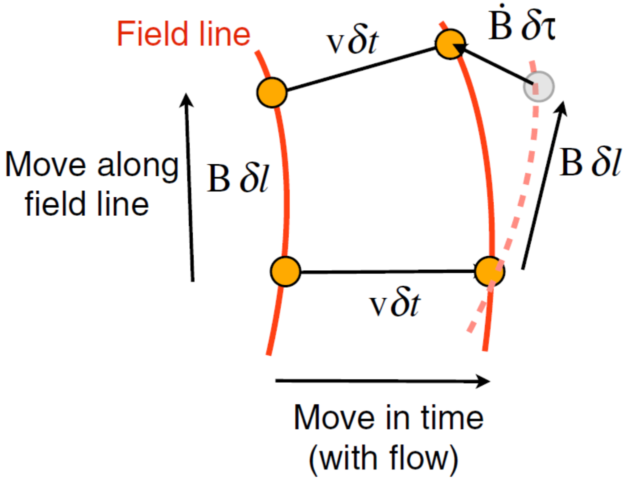


Introducing Nulness

- Bateman's construction (1915); also, Penrose twister theory (1967):

$$\mathbf{E} \cdot \mathbf{B} = 0, \qquad \mathbf{E} \cdot \mathbf{E} - \mathbf{B} \cdot \mathbf{B} = 0$$

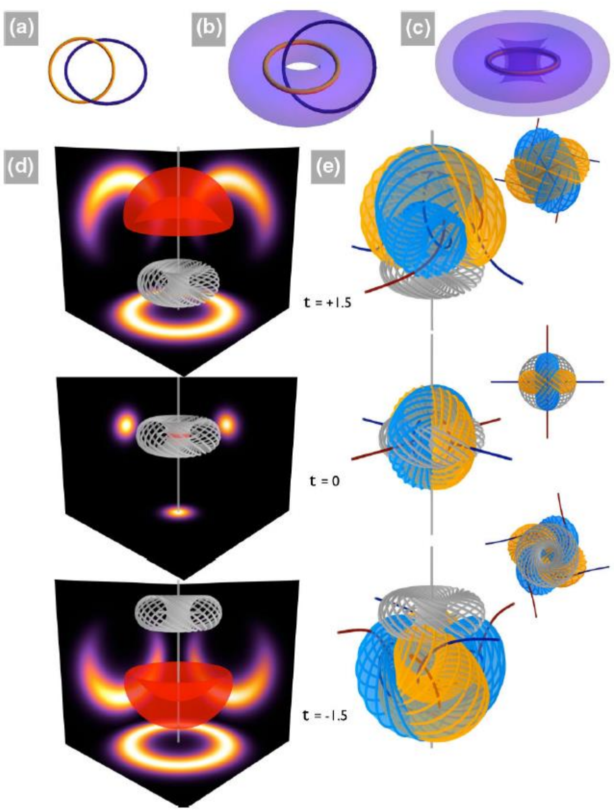
At t = 0 each of the electric, magnetic, and Poynting field lines have identical structure (that of Hopf fibration), oriented in space so they are mutually orthogonal to each other.



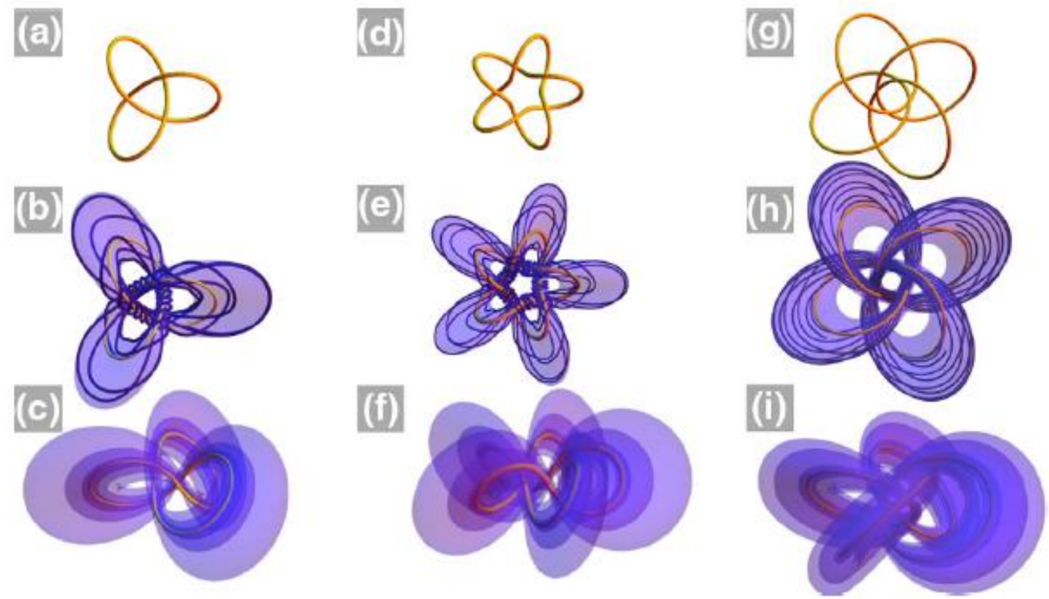
- For a null electromagnetic field, the Poynting vector not only guides the flow of energy, but also governs the evolution of the electric and magnetic field.
- This constraint introduces a non-linearity that can preserve the soliton-like excitation from dissipation with time.

The new family of Hopfion solutions that do not dissipate with time

The new result of Hridesh Kedia et al. (2013):



Hopfion solution: (a) – (c) Field line structure, (d) – (e) Field line nested tori forming closed loops linked with every other loop



Structure of magnetic field lines:
(a) – (c) Trefoil knots,
(d) – (f) Cinquefoil knots,
(g) – (i) 4 linked rings

The new family of Hopfion solutions that do not dissipate with time [Hridesh Kedia et al. (2013)]

Time evolution of magnetic field and energy density tor the (a) trefoil knot, (b) the cinquefoil knot and © the 4-Hopf linked rings